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One Approach to the Time-Optimal Strategy Formulation for Analog Circuit Design

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Abstract

The formulation of the process of analog system design has been done on the basis of the control theory application. This approach produces many different design strategies inside the same optimization procedure and allows determining the problem of the optimal design strategy existence from the computer time point of view. Different kinds of system design strategies have been evaluated from the operations number. This analysis shows that the traditional approach is not time-optimal at least for the electronic circuit design. General methodology for any analog system design was formulated by means of the optimum control theory. The main equations for this design methodology include the special control functions that are introduced to generalize the design process. The problem of the time-optimal design algorithm construction is defined as the minimal-time problem of the control theory. Numerical results of some nonlinear passive and active electronic circuit design demonstrate the efficiency of the proposed methodology.

Keywords:

Time-optimal design strategy, control theory application, minimum-time problem.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement and the fundamental technology trends underscore the need to rethink conventional design methodology for the system design (Costello, 2001). This problem has a special significance for the VLSI electronic circuit design. Any system design methodology includes two main parts as a rule: the model of the system which can be simulated as algebraic equations or differential-integral equations and a parametric optimization procedure that achieves the objective function optimal point. The traditional design strategy for the system design has two fixed determined parts. The first part is the mathematical model of the physical system and the second one is the optimization procedure. In limits of this conception it is possible to change optimization strategy and use different models and different analysis methods. However, the time of the large-scale circuit analysis and the time of optimization procedure increase when the network scale increases.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose (Bunch and Rose, 1976; Duff and Reid, 1979; Osterby and Zlatev, 1983; George, 1984). Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in (Wu, 1976), or by nodes tearing as in (Sangiovanni-Vincentelli et al., 1977) and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation (Rabat et al., 1985). Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in (Ruehli et al., 1982; Ruehli and Ditlow, 1983) for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization (Gill et al., 1981; Fletcher, 1981). The practical aspects of use of these methods are developed for VLSI circuit design, yield, timing and area optimization (Brayton and Hachtel, 1981; Ruehli, 1987; Massara, 1991). It is possible to suppose that the circuit analysis methods and the optimization procedures will be improved later on. Meanwhile, it is possible to reformulate the total design problem and generalize it to obtain a set of different design strategies inside the same optimization procedure. It is clear that a finite but a large number of different strategies include more possibilities for the selection of one or several design strategies that are time-optimal or quasi-time-optimal ones. This is especially right if we have infinite number of the different design strategies. On the contrary of the traditional design strategy, the modified traditional design strategy has only one part, because all system parameters are determined as independent and the objective function of the optimization procedure includes additional penalty functions, that describe the model of the physical system. In this case the equations of the model of the physical system disappear. On the other hand, it is possible to re-determine the total design problem, to generalize it, to obtain a set of the different design strategies. First of all, we define the time-optimal design strategy as the algorithm that achieves the optimum point of the objective function of the design process at the minimal computer time. The main problem of this formulation is the search of the special conditions, which need to be satisfied for the optimal algorithm construction.

The idea of the control theory use, which was introduced in (Zemliak, 1999) is developed now for the design of the systems that are described by the non-linear algebraic equation model. This methodology generalizes the design problem and can reduce the total necessary computer design time. First of all the evaluation of the operations number for different design strategies has been done. The main system of equations that describes the general design process is determined. This system includes the special control functions, which generalize the design process. The time-optimal system design procedure is defined as a minimal-time problem of the control theory and gives the possibility to use the specific methods of this theory. Different examples of the electronic circuit design have been solved using the proposed methodology.

2 The Problem Formulation

The design process for any analog system design can be defined as the problem of the objective function $C(X)$ minimization for $X \in R^N$ with the system of constraints. It is supposed that the minimum of the objective function $C(X)$ achieves all design objects and the constraint system is the mathematical model of the physical system. It is supposed also, that the system model can be described as the system of nonlinear equations:

$$g_j(X) = 0 \qquad (1)$$

$$j = 1, 2, \ldots, M$$

The vector X can be separated in two parts: $X = (X', X'')$. The vector $X' \in R^K$ can be named as the vector of independent variables, where K is the number of independent variables, and the vector $X'' \in E^M$ is the vector of dependent variables, where $N = K + M$. This separation is very conditional, because any variable can be defined as independent or dependent parameter. If the electronic system is described, it is more traditional and natural to define the system elements as independent variables and the physical parameters (voltages, currents, and so on) as dependent variables, but it is not obligatory.

The optimization process for the objective function $C(X)$ minimization for two-step procedure can be defined in general case as following vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \qquad (2)$$

with constraints (1), where s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, H is the direction of the objective function $C(X)$ decreasing. The vector H is the function of $C(X)$. This is a typical formulation for the constrained optimization problem. This problem can be transformed to the unconstrained optimization problem for $K = N - M$ variables. In this case the design problem is defined in more traditional form as an unconstrained optimization process in the space R^K:

$$X'^{s+1} = X'^s + t_s \cdot H^s \qquad (3)$$

with the system (1) which is solved at each step of the optimization procedure.

The specific character of the design process at least for the electronic systems consists in the fact that it is not necessary to fulfill the condition (1) for all steps of the optimization process. It is quite enough to fulfill these conditions for the final point of the design process.

The problem (1), (3) can be redefined in form when there is no difference between independent and dependent variables. All components of the vector X can be defined as independent. This is the main idea for the penalty function method application. In this case the vector function H is the function of the objective function $C(X)$ and the additional penalty function $\varphi(X)$: $H^s = f\left(C(X^s), \varphi(X^s)\right)$. The penalty function structure includes all equations of the system (1) and can be defined for example as:

$$\varphi(X^s) = \frac{1}{\varepsilon}\sum_{i=1}^{M} g_i^2(X^s) \tag{4}$$

In this case we define the design problem as the unconstrained optimization (2) in the space R^N without any additional system but for the other type of the objective function $F(X)$. This function can be defined for example as an additive function:

$$F(X) = C(X) + \varphi(X).$$

In this case we achieve the minimum of the initial objective function $C(X)$ and comply with the system (1) in the final point of the optimization process (in the minimal point of the function $F(X)$). This method can be named as modified traditional design method and it produces another design strategy and another trajectory line in the space R^N.

On the other hand, it is possible to generalize the idea of the additional penalty function application if to make up the penalty function as one part of the system (1) only, and the other part of this system is defined as constraints. In this case the penalty function includes first Z items only, $\varphi(X^s) = \frac{1}{\varepsilon}\sum_{i=1}^{Z} g_i^2(X^s)$ where $Z \in [0, M]$ and M-Z equations make up one modification of the system (1):

$$g_j(X) = 0 \tag{1'}$$

$$j = Z+1, Z+2, \ldots, M$$

It is clear, that each new value of the parameter Z produces a new design strategy and a new trajectory line. This idea can be generalized more (Zemliak, 1999) in case when the penalty function $\varphi(X)$ includes Z arbitrary equations from the system (1). The total number of different design strategies is equal to 2^M, if $Z \in [0, M]$. All these strategies exist inside the same optimization procedure. The optimization procedure is realized in the space R^{K+Z}. The number of the dependent variables M increases rapidly with the system complexity increasing. In this case the number of different design strategies increases exponentially. It is clear that these different strategies have various computer times because they have different operations number. It is appropriate in this case to define the problem of the search of an optimal design strategy that has the minimal computer time. Here and further the design strategy optimality is defined as the computer time minimization.

The most general approach can be constructed on the basis of the design problem formulation as the problem of optimal control (Zemliak 1999, 2001). It is possible to define a design strategy by equations (1'), (2) with the variable value of the parameter Z during the optimization process. It means that we can change the number of independent variables and the number of the terms of the penalty function at each point of the optimization procedure. It is convenient to introduce in consideration a vector of the special control functions $U = (u_1, u_2, \ldots, u_M)$ for this aim, where $u_j \in \Omega$; $\Omega = \{0;1\}$. These control variables are introduced artificially to generalize the design process. The sense of the control function u_j is next: the equation number j is present in the system (1') and the term $g_j^2(X)$ is removed from the right part of the formula (4) when $u_j = 0$, and on the contrary, the equation number j is removed from the system (1') and is present in the right part of the formula (4) when $u_j = 1$. In this case we have the following formulas for the model of the system and for the penalty function:

$$(1 - u_j)\, g_j(X) = 0 \tag{5}$$

$$j = 1, 2, \ldots, M$$

$$\varphi(X^s) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2(X^s) \tag{6}$$

All control variables u_j are the functions of the current point of the optimization process. The vector of the directional movement H is the function of the vectors X and U in this case: $H = f(X, U)$. The total number of the different design strategies, which are produced inside the same optimization procedure, is practically infinite. Among all of these strategies exist one or few optimal strategies that achieve the design objects for the minimum computer time. So, the problem of the time-optimal design strategy finding is formulated as the typical minimal-time problem of the control theory. The main problem of this definition is unknown optimal dependencies of all control functions. The solution of this problem may be find by some approximate methods of the optimal control theory.

There are many well-developed methods of the unconstrained optimization for the function of the multiple arguments. We need to elect an optimization procedure for further evaluation and analysis. Almost all optimization methods can be classified on zero order methods, as method of dual directions, method of conjugate directions, Hooke-Jeeves method (Hooke and Jeeves, 1961), simplex method and so on; the first order methods as the different variants of the gradient method; the second order methods as the different variants of the Newton's method; and the quasi-Newton methods. These last methods are based on the approximation of the matrix of second derivatives and include the method of conjugate gradients, Fletcher-Reeves method (Fletcher and Reeves, 1961), Davidon-Fletcher-Powell method (Fletcher and Powell, 1963) and so on.

It is convenient to select the gradient method for the optimization procedure to simplify a further analysis. This method has some defects of the convergence, but serves well as the basis for many other algorithms. On the other hand, the gradient method with its modifications appears practically in all optimization methods, which use the objective function derivatives. Finally, the gradient method allows revealing the main features of the new design problem formulation.

It is possible to reformulate the optimization process (2) on continuous form by means of following differential equation:

$$\frac{dX}{dt} = f(X,U)$$

It means, that the main problem of the design process can be formulated as the problem of the integration of this system with additional condition (5). The structure of the function H for the gradient method is given by:

$$H \equiv f(X,U) = -\frac{\P F(X,U)}{\P X}$$

This function defines the direction of the movement during the optimization process.

3 Operations Number Evaluation

3.1 The traditional design strategy

The traditional design strategy includes two systems of equations. It is supposed that the optimization procedure for the system design process can be defined as the system of the ordinary differential equations for the independent variables, for example as:

$$\frac{dx_i}{dt} = -b \cdot \frac{d}{dx_i} C(X) \tag{7}$$

$$i = 1,2,\dots,K$$

where $C(X)$ is the objective function of the design problem; b is the iteration parameter; the operator $\frac{d}{dx_i}$ hear and below means $\frac{d}{dx_i} j\,(X) = \frac{\P j\,(X)}{\P x_i} + \sum_{p=K+1}^{K+M} \frac{\P j\,(X)}{\P x_p} \frac{\P x_p}{\P x_i}$. To simplify the analysis, the gradient method is used as the optimization procedure here and below. However, it is not important what kind of the optimization method is used. It is only necessary to prepare the optimization procedure as the system of ordinary differential equations for the independent variables.

The model of the system is determined as the system of constraints. It is supposed also, that this model is described as the system of the non-linear algebraic equations:

$$g_j(X) = 0 \tag{8}$$

$$j = 1,2,\dots,M$$

The operations number for the solution of the system (8) by the Newton's method is equal to $S \cdot [M^3 + M^2(1+P) + MP]$, where P is the average operations number for the function $g_j(X)$ calculation; S is the iteration number of Newton's method for the system (8) solution. In the case when the quasi-Newton method is used it is necessary to change this formula to another: $S' \cdot (M^2 P + MP)$ where S' is the iteration number of quasi-Newton method ($S' > S$). The operations number for the one step integration of the system (7) by the Newton's method is equal to $K + C \cdot (1+K) + (1+K) \cdot S \cdot [M^3 + M^2(1+P) + MP]$, where C is the operations number for the objective function calculation. The total operations number for the solution of the problem (7) - (8), when the Newton's method is used is equal to:

$$N_1 = L_1\{K + (1+K)\{C + S \cdot [M^3 + M^2(1+P) + MP]\}\} \tag{9}$$

where L_1 is the total steps number of the optimization algorithm. The Newton's method for the solution of the system (8) was taken into consideration below to evaluate the total operations number. The results for the quasi-Newton method are very similar.

3.2 The modified traditional design strategy

The modified traditional strategy is determined as the system of optimization procedure equations without any constraints. In this case the number of independent variables is equal to $K+M$. The principal system is given by

$$\frac{dx_i}{dt} = -b \cdot \frac{d}{dx_i} F(X) \tag{10}$$

$$i = 1,2,\ldots, K+M$$

where $F(X)$ is the general objective function, $F(X) = C(X) + \frac{1}{e} \sum_{j=1}^{M} g_j^2(X)$. The total operations number for the problem (10) solution is equal to:

$$N_2 = L_2\{K+M+(1+K+M)\cdot[C+(P+1)M]\} \tag{11}$$

3.3 The general design strategy

It is possible to define the general design strategy as the strategy which has the variable number of independent parameters, that is equal to $K+Z$. In this case the following two systems are used:

$$\frac{dx_i}{dt} = -b \cdot \frac{d}{dx_i} F(X) \tag{12}$$

$$i = 1,2,\ldots, K+Z$$

and

$$g_j(X) = 0 \tag{13}$$

$$i = Z+1, Z+2, \ldots, M$$

where $F(X) = C(X) + \frac{1}{e} \sum_{j=1}^{Z} g_j^2(X)$. In this case the total operations number N_3 for the solution of the systems (12), (13) is equal to:

$$N_3 = L_3\{K+Z+(1+K+Z)[C+(P+1)Z+ S\cdot[(M-Z)^3+(M-Z)^2(1+P)+(M-Z)P]]\} \tag{14}$$

This formula is turned to the formula (9) when $Z=0$ and is turned to the formula (11) when $Z=M$. Analysis of the operations number N_3 as the function of Z gives the conditions for the minimal computer time calculation. This general strategy almost has no preference in computer time when the system (13) is linear or quasi-linear. In this case the iteration number for the Newton's method S is equal to 1 and the traditional approach is optimal. It is supposed also that the iterations number L_3 and the operations number C for the objective function calculation have dependencies from the independent variables' number by the following law:

$$L_3 = L_0 \cdot (K+Z)^n; \quad C = C_0 \cdot (K+Z)^m .$$

These are ordinary assumptions and the principal problem is the value of the power n and m. On the other hand, the iterations number S for the Newton's method don't has dependency from the order of the system (13) in the first approximation and is equal to constant value S_0. This value in practical situation is equal to 4 - 5 to achieve the precision $d = 10^{-10} - 10^{-12}$. The average operations number P for the function $g_j(X)$ calculation has no dependency from Z, if the electronic circuit is analyzed. This is correct because the admittance matrix of the electronic circuit is very sparse. It is supposed that this value is constant and equal to P_0. In this case, the formula (14) for the function $N_3(Z)$ calculation is transformed to:

$$N_3(Z) = L_0\cdot(K+Z)^n\cdot\{K+Z+(1+K+Z)[C_0\cdot(K+Z)^m+Z(1+P_0)] +S_0\cdot((M-Z)^3+(M-Z)^2(1+P_0)+(M-Z)P_0)]\} \tag{15}$$

In accordance with the principal definition of the optimal design strategy we can find the optimal strategy by the analysis of this formula. We need to find the optimum point Z_{opt}, where the function $N_3(Z)$ has the minimum value. If the optimum point Z_{opt} is equal to 0 it means that the traditional strategy is the optimum one. If the optimum point Z_{opt} is equal to M it means that the modified traditional strategy is the optimum one. If the optimum point Z_{opt} belongs to the region (0, M), it means that one of the intermediate strategies is the optimum one. The derivative of the function $N_3(Z)$ is given by the formula:

$$N_3'(Z) = L_0 n(K+Z)^{n-1}\{K+Z+(1+K+Z)[C_0(K+M)^m + Z(1+P_0)+S_0((M-Z)^3+(M-Z)^2(1+P_0)+(M-Z)P_0)]\} + L_0(K+Z)^n\{1+C_0(K+M)^m+(1+K+2Z)(1+P_0)+S_0[(M-Z)^3 + (M-Z)^2(1+P_0)+(M-Z)P_0 - (1+K+Z)(3(M-Z)^2+2(M-Z)(1+P_0)+P_0)]\} \tag{16}$$

To obtain the minimum point as an inside point of this region [*0, M*] it is necessary to provide two conditions for the derivative in the boundaries: $N_3'(0)<0$ and $N_3'(M)>0$. The derivative $N_3'(0)$ in assumption that $m = 1$, for the point $Z = 0$ is given by formula:

$$N_3'(0) = L_0K^n\{(1+n)[1+C_0(K+M)+S_0(M^3+M^2(1+P_0) + MP_0]+K(1+P_0)-S_0K[3M^2+2M(1+P_0)+P_0]\} =$$

$$= L_0K^{n+1}M^2\left\{(1+n)\left[\frac{1}{KM^2}+\frac{C_0(K+M)}{KM^2}+S_0\left(\frac{M}{K}+\frac{1+P_0}{K}+\frac{P_0}{KM}\right)\right]\right\} + L_0K^{n+1}M^2\left\{\frac{1+P_0}{M^2}-S_0\left[3+\frac{2(1+P_0)}{M}+\frac{P_0}{M^2}\right]\right\} \tag{17}$$

It is convenient to define an additional parameter $q = \frac{M}{K}$. The formula (17) is transformed, when $M, K \to \infty$, to the following formula: $N_3'(0) = L_0K^{n+1}M^2S_0\left[(1+n)q-3\right]$. It is necessary to provide a special condition for the parameter n to fulfill the condition $N_3'(0)<0$. This condition is given by the formula $n < \frac{3}{q} - 1$. Parameter q for the majority of the systems is less than or equal to 1. In that case we have the condition for the parameter n as: $n < 2 + e$.

On the other hand, the derivative $N_3'(Z)$ in the point $Z=M$ has the following form:

$$N_3'(M) = L_0(K+M)^n\{(1+n)[1+C_0(K+M)]+nM(1+P_0) +(1+K+2M)(1+P_0)-S_0P_0(1+K+M)\} =$$

$$= L_0(K+M)^{n+1}\left\{\frac{1+n}{K+M}+C_0(1+n)+\frac{nM(1+P_0)}{K+M}\right\} + L_0(K+M)^{n+1}\left\{\frac{(1+K+2M)(1+P_0)}{K+M}-S_0P_0\right\} \tag{18}$$

This formula is transformed, when $M, K \to \infty$, to the following form:

$$N_3'(M) = L_0(K+M)^{n+1}\left[C_0(1+n)+\frac{(1+K+2M+nM)(1+P_0)}{K+M}-S_0P_0\right].$$

It is supposed that the order n is equal to *2*. In that case the main inequality to provide the condition $N_3'(M)>0$ is given by

$$3C_0 + \frac{1+4q}{1+q}(1+P_0) - S_0P_0 > 0 .$$

This formula is transfomed, when $q \to 1$ and $C_0 \approx P_0$, to the following condition:

$$P_0(5.5 - S_0) + 2.5 > 0 .$$

In case when $n = 1$ another condition is given by

$$P_0(4 - S_0) + 2 > 0 .$$

There is a possibility to obtain the condition $N_3'(M)>0$ if the iteration number S_0 is equal to 4 or 5. Therefore, the optimum point Z_{opt} is within the region [*0, M*] in this case. This analysis serves as the basis for the subsequent more detailed investigation of the general design strategy idea.

The optimum point Z_{opt} for the problem (12), (13) minimizes the necessary computer time for the system design and, in general case, has dependency from the electronic system size and topology. This optimal point can be fined by different methods, for example by ordinary gradient method. The optimization of the independent parameters space dimension leads to the reduction of the total operation number and, therefore, to the reduction of the total computer time for the system design. In this work the problem of the optimum order of the space dimension is solved in a more general level by the optimal control theory approach. The total computer design time is served as the objective function for the time-optimal algorithm search. In that context the general design strategy can reduce the total computer time, but this strategy is not the most general because we have the constant number of the additional independent parameters Z_{opt} during the design process. It is possible to generalize more this strategy, if to change the number of the equations that are excluded from the system model (and are included to the objective function) at any moment of the design process. This idea was realized by the optimal control theory formulation.

4 System Design Time-Optimal Problem Formulation

It is possible to determine the problem of any analog system design as the problem of optimal control. The principal system of equations can be determined as:

$$\frac{dx_i}{dt} = f_i(X,U) \tag{19}$$

$$i = 0,1,\ldots,N$$

and

$$(1-u_j)g_j(X) = 0 \tag{20}$$

$$j = 1,2,\ldots,M$$

where N=K+M; x_0 is the additional variable; U is the vector of control variables, $U = (u_1, u_2, \ldots, u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.

The functions of the right hand part of the system (19) are determined as:

$$f_i(X,U) = -b\frac{d}{dx_i}\left\{C(X) + \frac{1}{e}\sum_{j=1}^{M} u_j g_j^2(X)\right\} \tag{21}$$

for $i = 1,2,\ldots,K$,

and

$$f_i(X,U) = -b\cdot u_{i-K}\frac{d}{dx_i}\left\{C(X) + \frac{1}{e}\sum_{j=1}^{M} u_j g_j^2(X)\right\} + \frac{(1-u_{i-K})}{dt}\left\{-x'_i + h_i(X)\right\} \tag{21'}$$

for $i = K+1, K+2, \ldots, N$,

where x'_i is equal to $x_i(t-dt)$; $h_i(X)$ is the implicit function $(x_i = h_i(X))$ that is determined by the system (20).

The sense of the control variables u_j is provided in section 2. These variables have the time dependency in general case. The equation number j is removed from (20) and the dependent variable x_{K+j} is transformed to the independent when u_j=1. This independent parameter is defined by the formulas (19), (21'). In this case there is no difference between formulas (21) and (21'), because the parameter x_{K+j} is an ordinary independent parameter. On the other hand, the equation (19) with the right part (21') is transformed to the identity $\frac{dx_i}{dt} = \frac{dx_i}{dt}$, when u_j = 0, because $h_i(X) - x'_i = x_i(t) - x_i(t-dt) = dx_i$. It means that at this time moment the parameter x_i is the dependent one and the current value of this parameter can be obtained from the system (20) directly. This transformation of the vectors X' and X'' can be done at any time. The function $f_0(X,U)$ is determined as the necessary calculation time for one step of the system (19) integration. The additional variable x_0 is determined as the total computer time T for the system design. In this case we determine the problem of the time-optimal system design as the classical minimal-time problem of the optimal control. In that context the aim of the optimal control is to take each function $f_i(X,U)$ to zero for the final time t_{fin} , $f_i\left(X(t_{fin}),U(t_{fin})\right) = 0$ and to minimize the total computer time x_0. By this formulation the general design strategy of the previous section is the particular case only. It is possible to re-determine this general design strategy as a method with the fixed values of all control functions u_j . The total number of the different design strategies, which is produced by the general design strategy, is equal to 2^M. On the contrary, the idea which defined the design process by means of equations (19)-(21) generates an infinite number of different design strategies. Each design strategy has its own trajectory in space R^N. It is clear, that the time comparison of the different trajectories is adequate only in case when the final trajectory point is the same. On the other hand, the objective function $C(X)$ has a set of local minimal points, because the design problem is a non-linear problem in general. It is necessary to put the additional simple conditions to achieve the same point of the objective function for the different design strategies. However, the non-simple problem is not a specific feature of new design problem formulation. We have this type of problem always when we begin the design process from different start points. It is supposed below that the simple conditions are provided.

To minimize the total design computer time it is necessary to find the optimal behavior of the control functions u_j during the design process. The functions $f_i(X,U)$ are not continued as the temporal functions in finite number of the time points because the control functions u_j have discontinuities. The minimal-time problem for the system (19) with the non-continued or non-smoothed functions (21),(21') can be solved most adequately by means of Pontryagin's maximum principle (Pontryagin *et al.*, 1962). For the classical Pontryagin's form of the optimal control problem formulation it is necessary to define the conjugate system for the additional functions ψ_i:

$$\frac{d\psi_i}{dt} = -\sum_{l=0}^{N} \frac{\partial f_l(X,U)}{\partial x_i} \cdot \psi_l \tag{22}$$

$i = 0,1,\dots,N$

Hamiltonian is determined as :

$$H(X,U,\Psi) = \sum_{i=0}^{N} \psi_i \; f_i(X,U) \tag{23}$$

This function has supreme value during the optimal trajectory with the Pontryagin's maximum principle:

$$M(X,\Psi) = \sup_{u \in \Omega} H(X,U,\Psi) \tag{24}$$

The main problem of the maximum principle application in that formulation is unknown vector Ψ_0 of the initial values of the functions ψ_i. This problem has adequate solution only for the linear functions $f_i(X,U)$, for example in (Neustadt, 1960). For the nonlinear case the direct application of the Pontryagin's maximum principle is very problematical. There are some iterative algorithms (Rosen, 1966; Tabak, and Kuo, 1969; Krylov and Chernousko, 1972; Fedorenko, 1978; Slotine and Li, 1991; Sepulchre et al., 1997) for the approximate solution of the problem (19)-(24). These algorithms are based on the boundary problem solution for $2\times(N+1)$ order equations system (19), (22). The iteration process for the numerical integration of this system includes consecutive iterations of Cauchy problem solution.

5 Examples

Some simple electronic circuits have been analyzed to demonstrate this system design approach based on the optimal control theory. All examples were divided in two groups. The circuits of the first group are passive nonlinear and the circuits of the second group are active nonlinear ones with transistors. The passive circuits have various nodal numbers from 1 to 5, $(M \in [1,5])$. Two examples of the transistor circuits have three and five nodes respectively. The design process has been realized on DC mode for all circuits. The detailed analysis of the passive electronic circuits for $M = 3, 4, 5$ is presented below in sections 5.1.1 – 5.1.3. The active circuit analysis is presented in sections 5.2.1, 5.2.2. The objective function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the nodal voltages for some nodes with additional inequalities for some circuit elements. It is supposed also that the additional physical constraints for the passive element are provided. All these elements are positive. To obtain this property it is convenient to change all admittance values y_i to x_i^2.

5.1 Passive nonlinear circuits

5.1.1 Example 1

In Fig. 1 there is a circuit that has four independent variables (*K=4*) as admittance y_1, y_2, y_3, y_4 and three dependent variables (*M=3*) as nodal voltages V_1, V_2, V_3 at the nodes 1, 2, 3.

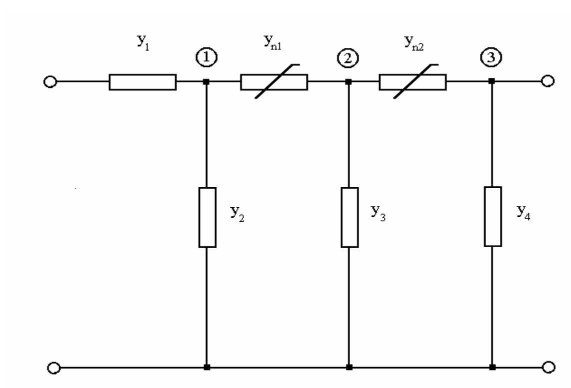

Figure 1. Circuit topology for 4 independent and 3 dependent parameters.

The non-linear elements are defined as $y_{n1} = a_{n1} + b_{n1}V_1^2$, $y_{n2} = a_{n2} + b_{n2}V_2^2$. The non-linearity parameters b_{n1}, b_{n2} are equal to 1.0. We define the components of the vector X by the formulas $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. In this case we have the system of seven differential equations as the optimization algorithm:

$$\frac{dx_i}{dt} = -b\frac{d}{dx_i}F(X,U)$$

i = 1,2,3,4

$$\frac{dx_i}{dt} = -b \cdot u_{i-4} \cdot \frac{d}{dx_i}F(X,U) + \frac{(1-u_{i-4})}{dt}\{-x_i(t-dt) + h_i(X)\}$$

i = 5,6,7

where

$$F(X,U) = C(X) + \frac{1}{e}\sum_{j=1}^{3} u_j g_j^2(x_1,x_2,x_3,x_4,x_5,x_6,x_7).$$

The model of the electronic system has three nonlinear algebraic equations in accordance with the nodal method.

$$g_1(X) \equiv (x_1^2 + x_2^2 + a_{n1} + b_{n1}x_6^2)x_5 - (a_{n1} + b_{n1}x_6^2)x_6 - x_1^2 = 0$$

$$g_2(X) \equiv -(a_{n1} + b_{n1}x_6^2)x_5 + (x_3^2 + a_{n1} + b_{n1}x_6^2 + a_{n2} + b_{n2}x_7^2)x_6 - (a_{n2} + b_{n2}x_7^2)x_7 = 0$$

$$g_3(X) \equiv -(a_{n2} + b_{n2}x_7^2)x_6 + (x_4^2 + a_{n2} + b_{n2}x_7^2)x_7 = 0$$

It is supposed that the input voltage is equal to 1. This system is transformed in accordance with our approach to the following system:

$$(1-u_j)g_j(x_1,x_2,x_3,x_4,x_5,x_6,x_7) = 0,$$

j = 1,2,3.

The results of the analysis of the complete set of the design strategies with the fixed value of the control functions are given in Table 1. There are eight different strategies in this case. The first line of the table corresponds to the traditional design strategy. The last line corresponds to the modified traditional strategy. The other lines correspond to the intermediate strategies.

N	Vector of the control functions U (u_1, u_2, u_3)	Iterations number	Total design time (sec)
1	(0 0 0)	394	0.21
2	(0 0 1)	9426	2.31
3	(0 1 0)	3038	0.77
4	(0 1 1)	8040	0.93
5	(1 0 0)	2178	0.66
6	(1 0 1)	6909	1.27
7	(1 1 0)	2810	0.44
8	(1 1 1)	8360	0.71

Table 1. Complete set of the design strategies for Example 1.

The total computer design time for the traditional design strategy in this case is equal to 0.21 sec. This is the optimal strategy among all the strategies, which were obtained with the fixed values of the control functions. However, this strategy is not optimal in general. It is necessary to find the optimal strategy by means of additional optimization procedure. Data of the time-optimal and some quasi-optimal strategies are given in Table 2.

N	Vector of the control functions U (u_1, u_2, u_3)	Switching points	Iterations number	Total design time (sec)
1	(1 1 0); (0 0 0)	100	412	0.177
2	(1 1 0); (0 0 0); (1 1 1)	100; 360	378	0.151
3	(1 1 1); (0 0 0)	180	416	0.142
4	(1 1 1); (0 0 0); (1 1 1)	180; 364	384	0.115

Table 2. Data of the optimum and quasi-optimum strategies for Example 2.

The strategy 4 is the optimum one and has the minimum computer design time that is equal to 0.115 sec. This strategy has two switching points and has the time gain 1.83 with respect to the traditional design strategy. The optimum behavior of the control functions u_1, u_2, u_3 during the total design process is shown in Fig. 2.

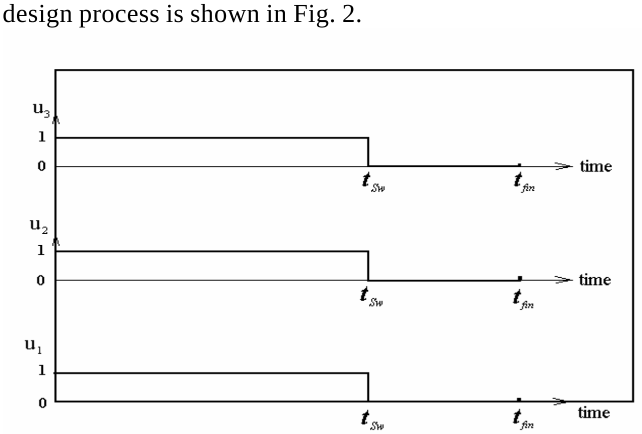

Figure 2. Optimum dependencies of the control functions u_1, u_2, u_3 *for Example 1.*

5.1.2 Example 2

In Fig. 3 there is a circuit that has five independent variables as admittance y_1, y_2, y_3, y_4, y_5 ($K=5$) and four dependent variables as nodal voltages V_1, V_2, V_3, V_4 ($M=4$) at the nodes 1, 2, 3, 4.

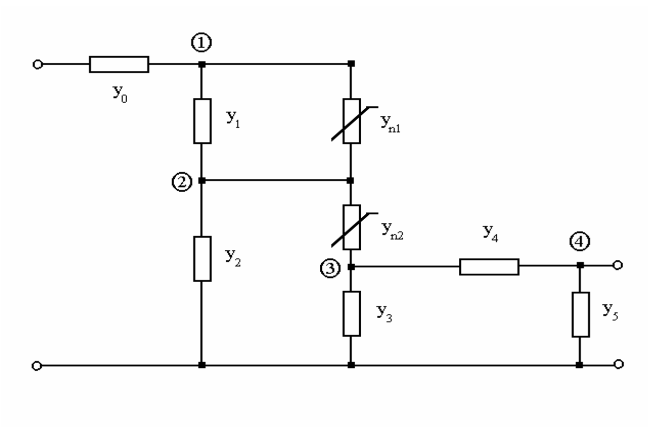

Figure 3. Circuit topology for 5 independent and 4 dependent parameters.

Non-linear elements have dependencies by the law: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2, \quad y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2.$ Non-linearity parameters b_{n1}, b_{n2} are equal to 1.0. The equations system of the optimization procedure and the model system equations have nine and four equations respectively. The results of the analysis of the complete set of the design strategies with the fixed value of the control functions are given in Table 3.

N	Vector of the control functions U (u_1, u_2, u_3, u_4)	Iterations number	Total design time (sec)
1	(0 0 0 0)	677	0.76
2	(0 0 0 1)	7412	5.01
3	(0 0 1 0)	1483	1.21
4	(0 0 1 1)	6434	2.31
5	(0 1 0 0)	1641	1.32
6	(0 1 0 1)	5785	2.03
7	(0 1 1 0)	2446	0.77
8	(0 1 1 1)	2426	0.55
9	(1 0 0 0)	742	0.61
10	(1 0 0 1)	5666	1.97
11	(1 0 1 0)	2205	0.77
12	(1 0 1 1)	5062	1.32
13	(1 1 0 0)	7563	2.42
14	(1 1 0 1)	24542	5.55
15	(1 1 1 0)	9244	1.42
16	(1 1 1 1)	6799	0.77

Table 3. Complete set of the design strategies for Example 2.

There are 16 different strategies in this case. It is interesting that among all of these strategies there are two strategies that have the design time less than the traditional design strategy. The strategy 8 has a design time that equals to 0.55 sec, but this strategy is not the optimal one either. The optimal trajectory was find by the special optimization procedure. The data of the optimal and some quasi-optimal strategies are given in Table 4.

N	Vector of the control functions U (u_1, u_2, u_3, u_4)	Switching points	Iterations number	Total design time (sec)
1	(0111);(0000)	150	346	0.2416
2	(1111);(0000)	400	550	0.2066
3	(0111);(1000)	150	355	0.1977
4	(1111);(1000)	450	600	0.1735
5	(0111);(0000);(1111)	150; 274	298	0.1683
6	(0111);(1000);(1111)	150; 277	309	0.1393
7	(1111);(0000);(1111)	400; 480	498	0.1347

Table 4. Data of the optimum and quasi-optimum design strategies for Example 2.

All strategies of this table have computer design time lesser than the best strategy 8 from Table 3. The strategy 7 is the optimum one. It has two switching points and has the minimal computer design time that is equal to 0.1347 sec. This strategy has the time gain 5.64 with respect to the traditional design strategy.

The optimum behavior of the control functions u_1, u_2, u_3, u_4 during the total design process is shown in Fig. 4.

Figure 4. Optimum dependencies of the control functions u_1, u_2, u_3, u_4 *for Example 2.*

These time dependencies define the minimal-time design procedure. These data show that it is impossible to determine the optimal behavior of the control functions without the special optimization procedure or the maximum principle.

5.1.3 Example 3

In Fig. 5 there is a circuit that has six independent variables as admittance $y_1, y_2, y_3, y_4, y_5, y_6$ ($K=6$) and five dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 ($M=5$) at the nodes 1, 2, 3, 4, 5.

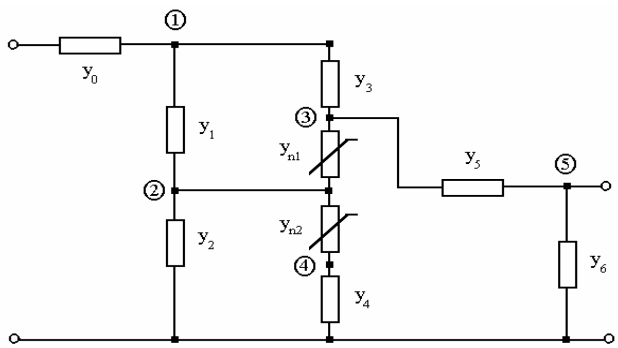

Figure 5. Circuit topology for 6 independent and 5 dependent parameters.

Non-linear circuit elements have dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_3 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_4 - V_2)^2$. Non-linearity parameters b_{n1}, b_{n2} are equal to 1.0. The system of the optimization procedure equations and the system of the model's equations have eleven and five equations respectively.

The results of the analysis of the complete set of the design strategies with the fixed value of the control functions are given in Table 5.

There are 32 different strategies in this case. There are three strategies among 32 different strategies that have the design time lesser than the traditional design strategy. The strategy 31 has the minimal design time that equals to 21.92 sec, but as for the previous examples this strategy is not optimal one either. The optimum trajectory was find by the special optimization procedure. Data of the optimum and some quasi-optimum strategies are given in Table 6.

All strategies of this table have computer design time lesser than the best strategy 31 from Table 5. The strategy number 9 is the optimal one. It has two switching points and has the minimal computer design time that is equal to 0.93 sec. This strategy has the time gain 32.4 with respect to the traditional design strategy.

N	Vector of the control functions U (u_1, u_2, u_3, u_4, u_5)	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	10165	30.11
2	(0 0 0 0 1)	28243	62.41
3	(0 0 0 1 0)	8134	22.68
4	(0 0 0 1 1)	201726	411.23
5	(0 0 1 0 0)	77216	218.27
6	(0 0 1 0 1)	340542	697.07
7	(0 0 1 1 0)	88238	177.68
8	(0 0 1 1 1)	408846	588.31
9	(0 1 0 0 0)	45726	155.71
10	(0 1 0 0 1)	270022	543.32
11	(0 1 0 1 0)	80502	161.43
12	(0 1 0 1 1)	561374	802.57
13	(0 1 1 0 0)	88747	218.71
14	(0 1 1 0 1)	493311	711.61
15	(0 1 1 1 0)	86338	52.18
16	(0 1 1 1 1)	568146	281.28
17	(1 0 0 0 0)	12146	26.81
18	(1 0 0 0 1)	82965	134.19
19	(1 0 0 1 0)	43318	87.27
20	(1 0 0 1 1)	251760	363.17
21	(1 0 1 0 0)	71611	145.77
22	(1 0 1 0 1)	355271	512.29
23	(1 0 1 1 0)	75043	106.78
24	(1 0 1 1 1)	401304	398.37
25	(1 1 0 0 0)	70004	170.65
26	(1 1 0 0 1)	392508	557.27
27	(1 1 0 1 0)	84754	121.17
28	(1 1 0 1 1)	564871	564.03
29	(1 1 1 0 0)	81863	140.94
30	(1 1 1 0 1)	471463	468.41
31	(1 1 1 1 0)	44249	21.92
32	(1 1 1 1 1)	634196	120.06

Table 5. Complete set of the design strategies for Example 3.

N	Vector of the control functions U (u_1, u_2, u_3, u_4, u_5)	Switching points	Iterations number	Total design time (sec)
1	(00110);(00000)	60	6510	19.22
2	(00010);(00000);(11111)	31; 5981	6047	17.74
3	(01010);(00000)	75	5510	16.25
4	(00110);(00000);(11111)	60; 5180	5236	15.32
5	(01110);(00000)	85	4463	13.07
6	(01010);(00000);(11111)	75; 4170	4230	12.36
7	(01110);(00000);(11111)	85; 3115	3185	9.06
8	(11110);(00000)	117	1748	4.88
9	(11110);(00000);(11111)	117; 403	467	0.93

Table 6. Data of the optimum and quasi-optimum design strategies for Example 3.

The optimum behavior of the control functions u_1, u_2, u_3, u_4, u_5 during the design process is shown in Fig. 6.

Figure 6. Optimum dependencies of the control functions u_1, u_2, u_3, u_4, u_5 for Example 3.

The optimum time dependencies of the control functions u_j for all examples have no any definite law and have been obtained by the special optimization procedure.

The results of all analyzed examples are the proof of that fact: the traditional design approach is not time-optimal. The comparison of these examples gives an important conclusion: the potential time gain that can be obtained by the above-described methodology increases when the system complexity grows. The computer time gain of the optimum design strategy with respect to the traditional design strategy as the function of the dependent parameters' number M is presented in Fig. 7. The time gain increases very fast with M increasing.

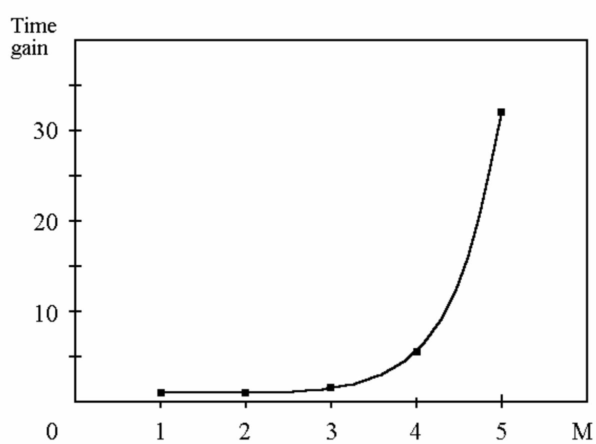

Figure 7. Computer time gain of the optimal design strategy.

5.2 Active nonlinear circuits

In Fig. 8 there is a circuit of the transistor amplifier that consists of two transistor cells.

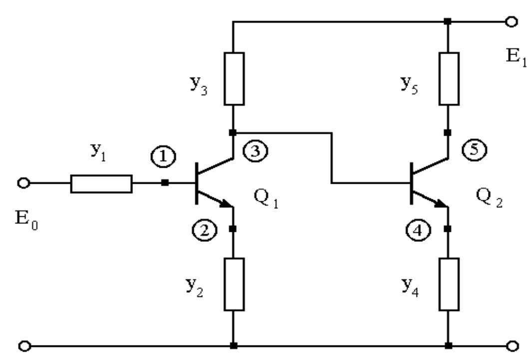

Figure 8. Circuit topology for two-cell transistor amplifier.

The one and two transistor cell circuits were analyzed separately. In the first case the circuit includes three nodes only *(M=3)*. The second circuit includes two transistor cells and has five nodes *(M=5)*. The design process has been realized on DC mode for both circuits. The Ebers-Moll static model of the transistor has been used. The objective function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the voltages for the transistor junctions.

5.2.1 Example 4

The one cell circuit has three independent variables as admittance y_1, y_2, y_3 (*K=3*) and three dependent variables as nodal voltages V_1, V_2, V_3 *(M=3)*. The results of the analysis of the complete set of the design strategies with the fixed value of the control functions and the optimal strategy are given in Table 7. The optimal strategy has two switching points and has time gain 16.2 with respect to the traditional design strategy.

N	Vector of the control functions U (u_1, u_2, u_3)	Iterations number	Switching points	Total design time (sec)
1	(0 0 0)	17748		45.15
2	(0 0 1)	59621		103.71
3	(0 1 0)	150136		282.53
4	(0 1 1)	67486		101.45
5	(1 0 0)	33770		56.58
6	(1 0 1)	7942		11.91
7	(1 1 0)	17623		25.93
8	(1 1 1)	37272		19.88
9	(1 1 0);(1 0 1); (1 1 1)	4280	540; 541	2.79

Table 7. Data of the one transistor cell circuit analysis.

5.2.2 Example 5

The two transistor cell circuit has five independent variables as admittance y_1, y_2, y_3, y_4, y_5 (*K=5*) and five dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 (*M=5*). The results of the analysis of the traditional design strategy, modified traditional strategy, some intermediate strategies with the fixed value of the control functions and the optimal design strategy are given in Table 8.

N	Vector of the control functions U (u_1, u_2, u_3, u_4, u_5)	Iterations number	Switching points	Total design time (sec)
1	(0 0 0 0 0)	33742		614.84
2	(0 0 1 0 0)	46766		702.44
3	(0 0 1 0 1)	28515		335.38
4	(0 0 1 1 0)	48575		568.64
5	(0 0 1 1 1)	40220		362.18
6	(0 1 1 0 0)	30133		357.62
7	(0 1 1 0 1)	14395		131.11
8	(0 1 1 1 0)	49256		446.43
9	(0 1 1 1 1)	20262		136.66
10	(1 0 1 0 0)	216520		2564.03
11	(1 0 1 0 1)	17535		102.93
12	(1 0 1 1 0)	181311		1638.14
13	(1 0 1 1 1)	18291		85.19
14	(1 1 1 0 0)	227083		2058.05
15	(1 1 1 0 1)	20289		94.58
16	(1 1 1 1 0)	125676		841.53
17	(1 1 1 1 1)	188522		234.53
18	(0 1 1 0 1); (1 1 1 1 1)	6380	3	7.9462

Table 8. Data of the two-transistor cell circuit analysis.

The optimal strategy has time gain 77.4 with respect to the traditional design strategy. This result confirms the rule, that the total computer time gain of the time-optimal design strategy increases when the complexity of the circuit increases. The comparison of the results for passive and active circuits shows that the computer time gain is larger for the active circuits because of more complexity in this last case. More operations number is required for the active circuits due to the exponential dependencies of nonlinear elements and owing to this a larger iteration number for all iteration processes.

The results of all analyzed examples show that the potential computer time gain of the time-optimal design strategy increases when the size and complexity of the circuit increase. This potential possibility exists due to practical infinite number of the different design strategies, which are included into the new design problem formulation. An additional optimization procedure is used for all analyzed examples, to construct the time-optimal trajectory. This practice serves well to prove the potential superiority of new approach, but it is not acceptable as the constructive searching method. The potential advantage is realized only in case when the current point of the design process moves along the optimal trajectory. For this case we need to construct the optimal algorithm systematically. This problem is open until the moment, but it is possible to search the solution for this problem on the basis of the approximate methods of the optimal control theory.

The above described approach serves as the theoretic foundation for the time-optimal design algorithm searching and promises to improve the design process characteristics when the optimal design algorithm will be constructed.

6 Conclusion

The traditional approach for the analog circuit design is not time-optimal. The problem of the optimum algorithm construction can be solved more adequately on the basis of the optimal control theory application. The time-optimal design algorithm is formulated as the minimal-time problem of the control theory. In this case it is necessary to elect one optimal trajectory from the quasi-infinite number of the different design strategies which are produced. The maximum principle can serve in this case as the basis for the election of the optimal dependencies of the control functions. This approach reduces considerably the total computer time for the system design. Analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal strategy increases when the size and complexity of the system increase. However, the problem of the time-optimal algorithm real construction is open. The above-described approach gives the possibility to find this algorithm as the approximate solution of the typical problem of the optimal control theory.

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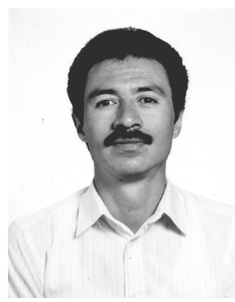

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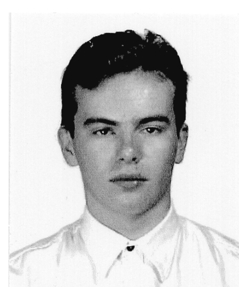

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