An Identification Genetic Algorithm for a Family of Duffing's System
Un Algoritmo Genético de Identificación para la Familia del Sistema de Duffing

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Abstract

This paper shows a simple way to recover the whole unknown parameters set of the Duffing's oscillator by using a genetic algorithm. The fact that the system is observable and constructible with respect to a suitable output helps in obtaining an integral parameterization of the output. Subsequently an integral parameterization of the output which depends upon the unknown parameters, and, a random estimation of the output is proposed, assuming that the set of unknown parameters are contained into a bounded set. This random estimation is chosen provided that the error between the actual output and the estimated output minimizes the errors of .a quadratic function. The minimization problem and the random estimations of the output are formulated directly in terms of a genetic algorithm. A population of chromosomes is codified with the parameters of the Duffing's oscillator system. A fitness function is established to evaluate the chromosomes, in such a way that it minimizes the errors of a quadratic function. The chromosomes` population evolves till a fitness average threshold is obtained. This method is numerically possible and easy to implement in a digital computer.
Keywords: Mechanical Oscillator, Chaos, Genetic Algorithms, Reconstruction.

Resumen

En este artículo se presenta una forma sencilla para estimar los parámetros desconocidos del oscilador de Duffing mediante el empleo de un algoritmo genético. El hecho de que el sistema es observable y construible con respecto a una salida disponible, ayuda a obtener una parametrización integral de la salida. A partir de esta parametrización se propone un estimador aleatorio de la salida, asumiendo que los parámetros desconocidos están contenidos en un conjunto acotado. El estimador aleatorio es propuesto de tal forma que el error entre la salida real y la salida estimada minimiza una función cuadrática. Así, el problema de minimización y del estimador aleatorio son resueltos mediante un algoritmo genético. La población de cromosomas es codificada con los parámetros del oscilador de Duffing. La función de adaptabilidad es establecida para evaluar los cromosomas, de tal forma que se minimice el error de la función cuadrática. Los cromosomas de la población evolucionan hasta que un umbral promedio de adaptabilidad es alcanzado. Este método es numéricamente posible y fácil de implantar en una computadora digital.
Palabras Clave: Oscilador Mecánico, Caos, Algoritmos Genéticos, Reconstrucción.

1. Introduction

The identification of chaotic attractors from one or more suitable variables is one of the most difficult problems in chaos theory and its applications. Several methods for identifying a chaotic systems have been proposed in the literature (for an in-depth treatment of those methods, the reader is referred to Chen, 1995; Middleton, 1990; and Huijbert *et al*., 2000). In these papers, the authors apply control theory to design state observers and system identification schemes for recovering the unknown parameters. The other important approach is based on the well-known Takens' Theorem (see Alligood *et al*., 1997; Sauer *et al*., 1991; Takens, 1981; Parlitz *et al*., 1994; and Makoto *et al*., 1997). This methodology consists in analyzing the observed time series from a nonlinear system to reconstruct a time delay of a phase space, in which it is possible to analyze the attractor. This is carried out by using time delayed values of an observed scalar quantity as coordinates for the phase space. Roughly speaking, vector state *y(n)* constructed as:

$$y(n) = \left[x(n), x(n+T),..., x(n+(d-1)T)\right]$$

can be estimated from a set of observations. Here x is the observed variable, T is the time delay and d is the embedding dimension (see Parker *et al.*, 1990). The last approach is based on soft computing, as proposed in (Poznyak *et al.*, 1998; Poznyak *et al.*, 1999; Yeap, 1994). In those papers, the unknown chaotic system is viewed as a black box belonging to a class of nonlinearities. Therefore, the dynamic neural network can be used to recover the unknown parameters.

In this paper, we show a simple and efficient approach for the revealing of all the unknown parameters and the estimation of the velocity state for the Duffing's system by using a genetic algorithm (**GA**). Loosely speaking, the role of a **GA** in any application is to evolve a chromosome population that codifies several possible solutions of the problem using genetic operators like *selection*, *crossover* and *mutation*. The goal of **GA** is the optimization of a fitness or cost function that depends on the problem to resolve. In our case, we must minimize the norm of a quadratic function that depends on the unknown parameters and successive integrations of a suitable output (the position of the Duffing's system). The integral parameterization of the output is necessary for making the positive function that will be minimized. The minimum of the function is reached when the actual parameter values are attained. The proposed methodology differs from the one described in Makoto *et al.* (1997) and Nayfeh (1979) because we discard the necessity of computing the derivatives of the measurable variable states (by using pairs of time-points and averaging the estimates), instead, we apply successive integrations of the output. It is worth to mention that the most common identification methods cannot estimate parameter w (which is the force frequency), while the one proposed in this paper successfully accomplishes the task.

The rest of the paper is organized as follows. Section 2 contains a brief introduction to Duffing's system. In Section 2 the observability property with respect to the precise output is discussed, therefore, an integral parameterization of the output can be obtained. Finally, in the same Section, we establish the framework for recovering the set of unknown parameters based on the minimization problem of some specific norm. Section 3 presents a brief and general description of **GA** and contains the numerical results, while Section 4 holds conclusions and suggestions for curious researchers and possible solutions to computing problems yet to be solved .
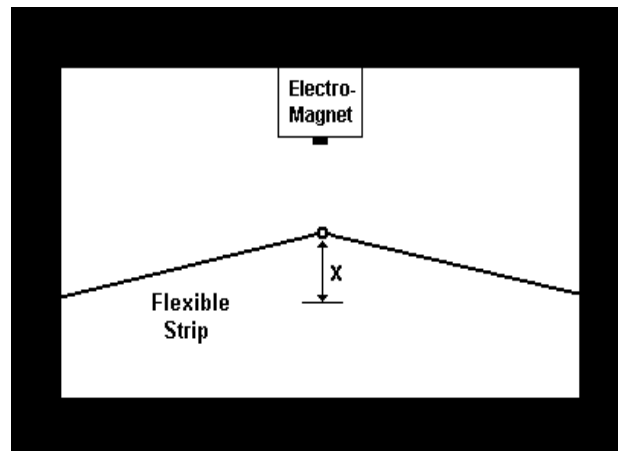

Figure 1: Duffing's oscillator

2 Duffing's Mechanical Oscillator

We consider a traditionally Duffing's mechanical oscillator, consisting of a light metal bar, hardly more than a thin strip, attached at each end by a solid support (see Fig. 1). Near the middle of the metal bar is an electro-magnet powered by alternating current. As the current in the magnet goes through a cycle, the magnetic field couples energy into the bar, forcing it to move. The movement is a flexing of the bar. As the bar in the oscillator flexes, the position of the midpoint is

used to measure the amount of movement. The non-linear model, which can be found in Alligood *et al.* (1997) and Kapitaniak (2000), is given by:

$$\dot{x} = v$$
$$\dot{v} = -p_1 v - p_3 x^3 + p_2 x + A\cos(wt) \qquad (1)$$

The vertical displacement of the midpoint of the flexible strip is measured by x. The magnitude of the forcing function is denoted with A, the forcing frequency is w, the damping coefficient is p_1, and the fixed constants, which are related to the non-linear stiffness function of the flexible strip, are p_2 and p_3.

It is known that the system has a chaotic behavior (see Parker, 1990) for the fixed values of parameters in a neighborhood of $p_1=0.4$, $p_2=-1.1$, $p_3=1$, $A=2.1$ and $w=1.8$.

2.1 An integral parameterization of the output

In this section, we introduce a simple integral parameterization for Duffing's system. This parameterization allows us to build a function that depends on the unknown parameters and on successive integrations of the measurable state (output).

Let us consider equation (1) and let us take as output $y = x$ (*i.e.* the position is known). Then, the velocity state $v = y$ can be obtained by means of an integral parameterization as follows:

$$v = v_0 - p_1(y - y_0) - p_3 \int y^3 + p_2 \int y \qquad (2)$$
$$+ \frac{A}{w}(\sin(wt) - \sin(wt_0)),$$

where y_0 and v_0 stand for the initial conditions of the states $x(t_0)$ and $v(t_0)$ respectively. And

$$\int x = \int_{t_0}^{t} x(t_1)dt_1 \ , \ \iint x = \int_{0}^{t}\int_{t_0}^{t_2} x(t_1)dt dt_2 \ .$$

Note that the integral parameterization is given by equation (2), and thus allows us to measure the velocity state v, as a function of the output, y modulo the initial conditions.

Integrating equation (2) once again with respect to time from initial time t_0 up to the final time t we have the following iterated integral equation:

$$y(t) = y_0 + v_0(t - t_0) + p_1\left[y_0(t - t_0) - \int y\right]$$
$$+ p_2 \iint y - p_3 \iint y^3 + g(A, w, t) \qquad (3)$$

and $g(A,w,t)$ stands for:

$$-\frac{A}{w}\left[\frac{1}{w}(\cos(wt) - \cos(wt_0)) + \sin(wt_0)(t - t_0)\right]$$

From (3) we say that the system is observable and constructible with respect to the output y (see: Martinez, 1996 and Sira 2002). The last equation contains the needed information for recovering the set of parameters. In the reconstruction process an error is generated, that must be minimized as a quadratic function of parameters $\{v_0, p_1, p_2, p_3, A, w\}$. In the following section we present such quadratic function.

2.2 Parameters identification for the Duffing's System

It is known that parameter recovering of a model based on the measurements of one or more variables leads to an optimization problem that is characterized by its ill-posed nature, in the Hadamard sense (see Hadamard, 1902, for more details), since a unique solution is impossible . Usually, the issue can be solved using some variant of Newton's method and either a conjugated gradient algorithm or even the least quadratic method (see Bender, 1999). So, we proceed to solve the problem as follows. First, a random parametric estimator for the available output is proposed. Based on that estimation, a quadratic error functions is formed, where the error is the difference between the actual output and the estimated one. This difference is computed in a discrete set of time. Finally, a simple **GA** is applied in the finding of an optimal solution.

We start with the identification problem for the Duffing's system:

Let $q=[\ q_0,\ q_1,\ q_2,\ q_3,\ q_4,\ q_5]$ be the vector of unknown parameters where $q_i \in R$ for $i = 0,1,...,5$, and let $\hat{y}(\cdot)$ be the following estimation output function:

$$\hat{y}(t,q) \equiv y_0 + q_0 f_1(t) + q_1\left[y_0 f_1(t) + f_2(t)\right]$$
$$+ q_2 f_3(t) + q_3 f_4(t) - q_4 f(q_4,q_5,t) \qquad (4)$$

Where the set of variables $\{\phi_1\,(t),\ \phi_2\,(t),\ \phi_3\,(t),\ \phi_4\,(t)\}$ is an iterated integral function of the measurable output, y defined by:

$$f_1(t) = t - t_0; \quad f_2(t) = -\int y;$$
$$f_3(t) = \iint y; \quad f_4(t) = -\iint y^3 \qquad (5)$$

And finally $f(A,w,t)$ denotes the time dependent function given by

$$f(q_4,q_5,t) = \frac{q_4}{q_5^2}\left(\cos(q_5 t) - 1\right)$$
$$+ \frac{q_4}{q_5}\sin(q_5 t_0)(t - t_0)$$

Now, consider the following estimation error defined as:

$$e(t) = y(t) - \hat{y}(t,q) \qquad (6)$$

Note that if vector q takes the real values of Duffing's system given by $q_\gamma = [v0,\ p_1,\ p_2,\ p_3,\ A,\ ]$ then $e(t) = 0$ for any time t. This means that the problem of finding the vector of unknown parameter $q \in R^6$ is clearly equivalent to solving the following unconstrained minimization problem:

$$\min_{q \in R^6} \sum_{k=1}^{n} e^2(kT) \qquad (7)$$

where T is the sampling time and n is the total number of samples.

To find the minimum of the last expression, it is necessary to introduce some basic assumptions:

A.1 The strings of outputs $y\,(t\text{-}k\ t)$ for a fixed delay $t> 0$ and $k=\{0,1,...,n\}$ are available for any time t, such that $t > k\,t$

A.2 The auxiliary functions $f_i(t\text{-}k\ t)$ for a fixed delay $t> 0$ and $k=\{0,1,...,n\}$ defined in (5) can be stored and computed.

A.3 The set of parameters of the non-linear system (1) are selected provided that their solution is bounded.

Notice that under assumption **A.3,** the solution of system (1) can exhibit a chaotic behavior for some particular sets of parameters. Also, when p_3 is equal to zero, it is well-known that system (1) becomes a periodical oscillator. It is worth observing that under assumption **A.**3 system (1) represents a wide variety of oscillator systems. For instance, the very common RLC circuit, the spring-mass system and the classical Duffing's oscillator.

Remark 1. The problem of finding q such that expression (7) is minimized, can be solved either by a numerical implementation of the well-known Newton's method or a variant of it. However, instead of Newton's method, we employ a **GA** which avoids the possibility of falling into a local minimum. The **GA** creates a population of q in a stochastic fashion according to some basic rules (see Goldber, 1989; Mitchel, 1998; and Bäck *et al.*, 2000). Then, it selects the element that produces the smallest error in (7).

Remark 2. The objective of all optimization problems is to find a minimum or maximum objective function value. Considering optimum values, usually a problem may have more than one optimum objective function value. There are many traditional deterministic algorithms available to solve optimization problems for a local minimum. Some of these methods include the descent gradient techniques. These methods require the evaluation of a gradient information in order to solve the problem. Gradient evaluations can become difficult and time consuming when complex objective functions are at sake . These methods always look for the closest minimum, without regarding it is a local or global one.

3. General description of GA to solve a minimization norm

Roughly speaking, the general framework to apply a **GA** can be summarized in the following six steps:

1. Individuals in the **GA** are vectors (in R6) of the form q i =[q0,i, q1,i, q2,i, q3,i, q4,i, q5,i]. It can be seen that the **GA** is real-coded (instead of a binary coded).

2. The initial population, P_0, contains *500* individuals, while subsequent populations, P_j, consist of *100* individuals. Thus allowing a wider search with the initial population and concentrating on more specific regions afterwards.

3. The best individual, q_1 (evidently ranked *1st*), in generation P_j is passed on to generation P_{j+1}, with no change.

4. Several steps are involved in the creation of generation P_{j+1}, they are: a) selection; b) crossover; c) mutation.

    a) To accomplish a selection, each individual in P_j is assigned a probability which is calculated linearly according to its ranking in the whole population . Selection of individuals is made by generating random numbers in *[0,1]* (say $á_i$) and comparing them to the accumulated probability, $Ap(q_i)$, of each individual. Individual q_i is selected to be part of P_{j+1} when $á_i \leq AP(q_i)$. This is the well-known *roulette selection* scheme.

    b) The crossover algorithm used in this **GA** is a slight modification of the *flat crossover* (or *arithmetic crossover*) operator (see Cardón, 2001; Bäck, 2000; and Michalewicz, 1999). An "offspring" $h = [h_0, h_1, h_2, h_3, h_4, h_5]$ is generated as

    $$h_i = \boldsymbol{b} \cdot q_{i-1} + (1 - \boldsymbol{b}) \cdot q_{i,2}$$

    From "parents"

    $$q_1 = [q_{0,1}, q_{1,1}, q_{2,1}, q_{3,1}, q_{4,1}, q_{5,1}]$$
    $$q_2 = [q_{0,2}, q_{1,2}, q_{2,2}, q_{3,2}, q_{4,2}, q_{5,2}]$$

    Where q_1 is a better individual that q_2 (i.e. q_1 makes the error function smaller than q_2 does).and $â$ is a random number chosen uniformly from the interval *[0.5,1]*. This interval is used in order to weight as more "influential" the information carried by the best of the parents. This process is repeated until there are *99* "offspring" (q_1 passes on unchanged).

c) The mutation algorithm consists into randomly changing an allele of *50%* of the individuals created at the last step. Changes are done within the vicinities specified below. This is the final step in creating generation P_{j+1}.

5. The "cost" of each individual was calculated via

$$\sum_{k=1}^{n} e^2(kT),$$

where T is the time sample. The algorithm stops when the best individual tags a "cost" named α, where α is fixed as small as needed.

6. Components of vector q=[q0, q1, q2, q3, q4, q5] were searched in a previously defined "box", it means

$$\left| q_i - q_{ic} \right| \leq e_i, \; i=1,2,...,5.$$

Where q_{ic} is the selected centre and e_i is the radius, which can be chosen as large as needed.

3.1. Numerical implementation of the GA

In order to evaluate the efficiency of the previously described method, computer simulations were usually carried out to estimate the unknown parameters *{p₁, p₂, p₃, A, w}* and initial state v_0 for the Duffing's system given in (1). The numerical program was implemented by using the fourth-order Runge-Kutta algorithm.

The computation was performed with a precision of *8* decimal digit numbers. To obtain a good performance, the step size in the numerical method was set to *0.001*. Parameter values were taken as *p₁=0.35, p₂=-1.014, p₃=0.957, A=2.15* and *w=1.893*. The sampling time was selected as *T=0.25* sec., the number of samples was chosen as *n=20*, the cost was selected $\alpha=\mathbf{10}^{-9}$ and the radius e_i was equal to *2.5* for every *i=1, 2,.., 5* and $e_0 = 5$.

The initial conditions where taken as $y(0)=0.3$ and $\dot{y}(0)=-2.3$ respectively. Finally, *q₀,c=0, q₁,c=0.1, q₂,c=-0.5, q₃,c=0.5, q₄,c=1,* and *q₅,c=1.2.*

In the following figures we present the obtained results after applying the previously described **GA**. In figure 2, four signals are displayed: the measurable output and three reconstructed signals.

These signals correspond to the best individual of generations *1, 200*, and *22793* (last generation). Note the absence of difference between the last one and   the original output. This means that the error in the estimated parameter is almost zero.
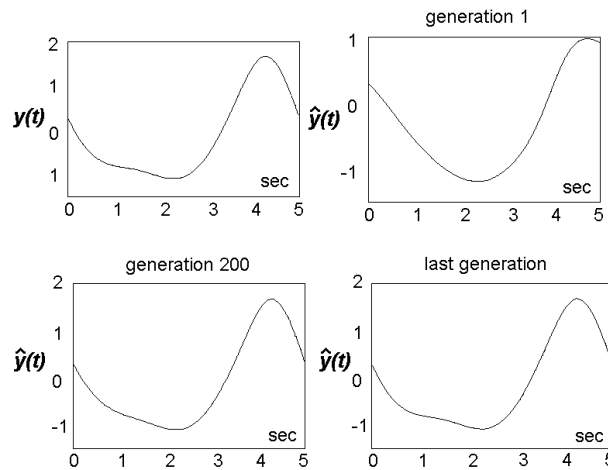

Figure 2: Actual output and three reconstructed signals.

Figures 3-5 show the evolution process of the initial state v_0 and of parameters p_1, p_2, p_3, A, w, respectively, through generations.
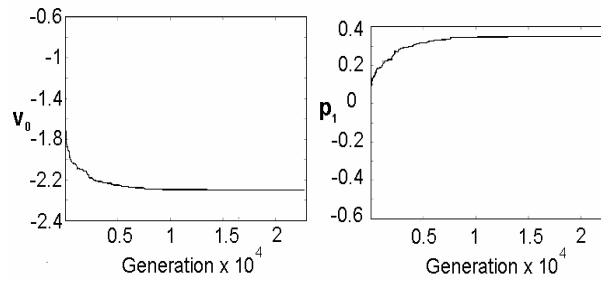

Figure 3: Evolution process of initial state v_0, and parameter p_1.
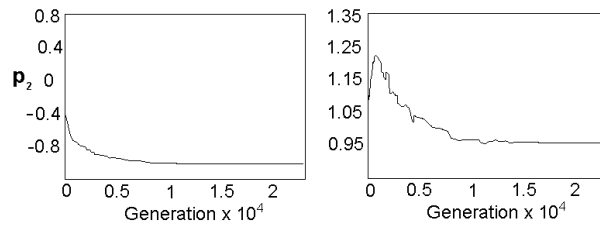

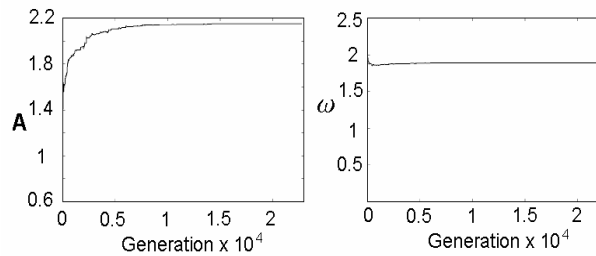**Figure 4**: Evolution process parameters p_2 and p_3.

Figure 5: Evolution process of parameters A and ù.

Finally, in Figure 6 we show the behavior of expression (7); the process of error minimization. It is fully understandable that the error tends to zero when the generation number increases, therefore, the reconstructed output is practically the original output. This explains why the actual and estimated parameters are very close, as shown in Table 1.
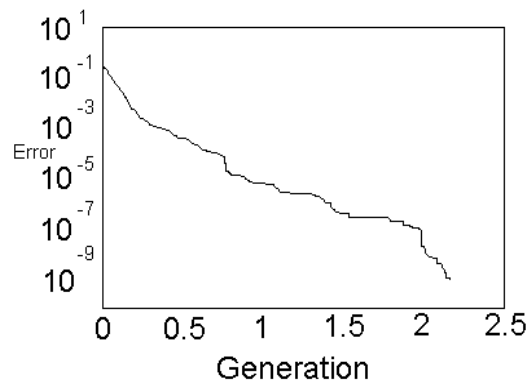

Figure 6: Error minimization process.

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Ge	p_1	p_2	p_3	A	ω
1	-0.570	0.631	0.882	0.665	0.346
3	-0.147	-0.360	0.975	1.259	2.215
9	0.158	-0.612	1.086	1.761	1.984
28	0.099	-0.633	1.077	1.499	1.964
86	0.124	-0.687	1.092	1.599	1.913
264	0.140	-0.707	1.15	1.675	1.879
804	0.184	-0.734	1.213	1.854	1.860
2451	0.269	-0.860	1.118	2.029	1.874
7475	0.335	-0.983	0.995	2.133	1.888
22793	0.350	-1.015	0.957	2.150	1.893

Table 1. Best individual of some generations.

4. Conclusions

A method for recovering parameters and estimating the velocity state of the Duffing's oscillator was proposed. We exploited the fact that the system is observable and constructible with respect to a measurable output. This property allowed us to build an iterated integral equation of the available output, which contains the required information for recovering the absent state and the unknown parameters. Based on the iterated integral equation, we estimated the output (defined in (4)) assuming that physical parameters of the system are contained in a bounded set. The basic idea is to minimize the average quadratic error, *i.e.*, the difference between the actual output and the estimated one, as described in (7). The minimization process is carried out by using the **GA**. This approach was validated by means of numerical experiments, in which the quadratic error was efficiently minimized. Therefore, the parameters and the unknown state could be satisfactorily estimated.

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