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Semiglobal Chaotification of One Class of *n*-Dimensional Continuous-Time Systems Via a Time-Delay Feedback

Caotificación Semiglobal de una Clase de Sistemas de n-Dimensiones de Tiempo Continuo a través Realimentación de Retardo

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Abstract

A semiglobal chaotification problem of *n*-dimensional polynomial continuous-time systems satisfying one special condition is examined. This problem is solved in two steps. Firstly, by using some polynomial mapping we transform the initial system to some *n*-th order explicit scalar ordinary differential equation with a rational nonlinearity. On the second step we apply the anticontrol feedback proposed by Wang, Chen and Yu. Our results are applied to nonlocal chaotification of three systems: the Lorenz system, the Burke-Shaw system and the Liu system.
Keywords: Polynomial systems, rational nonlinearity, chaos, anticontrol.

Resumen

En este artículo es examinado el problema de caotificación para sistemas polinomiales continuos de *n*-dimensiones satisfaciendo una condición especial. Este problema es resuelto en dos pasos. Primero, a través del uso de un mapeo polinomial, se transforma el sistema inicial en una ecuación diferencial ordinaria escalar de orden n con una no linealidad racional. En el segundo paso se aplica el anticontrol por retroalimentación propuesto por Wang, Chen y Yu. Los resultados de caotificación no local son aplicados a tres sistemas, el sistema de Lorenz, el sistema de Burke-Shaw y el sistema de Liu.
Palabras clave: Sistemas polinomiales, no linealidad racional, caos, anticontrol.

1 Introduction

Control of chaos has received considerable interest during the last ten years. The corresponding literature is very vast and contains many papers, see e.g. books on this area (Chen et al., 1998; Chen, 1999; Fradkov et al., 1999; Judd et al., 1997). Contrary to this, anticontrol of chaos is a topic being investigated intensively only since the end of nineties. The term "anticontrol of chaos" means chaotification of a dynamical system, i.e. creating chaos in a system by introducing a proper controller. Now it is well- recognized that chaos is beneficial in some important areas like as secure communication, liquid mixing, information processing and others. In the most of papers chaotification of discrete-time systems were studied. Local chaotification of nonlinear continuous-time systems via a time-delay feedback was considered in (Wang and Chen, 2000; Wang and Chen, 2000; Wang et al., 2000). Chaotification around the limit cycle was described in (Chen and Wang, 2003). Global chaotification problem for nonlinear continuous-time systems via a time-delay feedback was examined in (Starkov and Chen, 2004) and in (Chen et al., 2004). Other related references are (Chen et al., 2002; Tang et al., 2003; Yang et al., 2002). In papers cited here one can find also references on chaotification of discrete-time systems. In this paper we examine the *semiglobal chaotification problem* for polynomial systems with applications to one class of three-dimensional systems with a quadratic right side. Comparing with (Starkov and Chen, 2004), we apply here another way of transforming the initial system to some *n*-th order explicit scalar ordinary differential equation with a rational nonlinearity. This paper is a reworked and enlarged version of the paper (Starkov, unpublished). Our approach is illustrated by examples of the Lorenz system, the Burke- Shaw system and the Liu system. Results of numerical simulation are provided as well.

2 Preliminary on chaotification of continuous-time systems by a time-delay feedback

In (Wang et al., 2000) Wang, Chen and Yu suggested one chaotification method of stable linear systems and its application to nonlinear systems possessing a locally stable equilibrium. Their method allows to create chaos in an originally nonchaotic system and to enhance areas of parameters values for which a chaotic system remains chaotic. In briefly, their idea is as follows. They consider the *n*-th order single-input linear time-invariant ordinary differential equation: $x^{(n)}(t)+a_{n-1}x^{(n-1)}+... +a_1x^{(1)}(t)+a_0x(t)=\beta_0u(t),$ with a Hurwitz system matrix. It is shown that the map $x_{k+1}=\varepsilon_0\sin(\xi x_k),$ with $\varepsilon_0>0$ be given, is chaotic for sufficiently large ξ, and in this case the choice of a time-delay feedback $u=\varepsilon\sin(\xi x(t-\tau))$ makes this *n*-th order differential equation chaotic for a sufficiently large τ; here $\varepsilon_0=\varepsilon\beta_0/a_0$. Then by application of local exact linearization condition for a locally stable linear-in-control system and this time-delay feedback they get chaotification. The neighborhood of a locally stable equilibrium where the chaos is obtained by this approach is not described explicitly. So one can talk that this is the case of local chaotification. Our main contribution concerns nonlocal chaotification of polynomial systems satisfying some special condition called the invertibility condition; this chaotification in some cases is occurred to be global. In addition of using this time-delay feedback, the underlying idea is related to the fact that a system of the type considered can be mapped into a system which can be written as a *n*-th order explicit scalar ordinary differential equation with a rational nonlinearity.

3 Semiglobal chaotification of one class of polynomial systems

Firstly, let us take a polynomial system $\sum_n:\dot{x}=f(x)+ug(x),x\in\mathbf{R}^n,$ and any real polynomial $h(x)$ of *n* variables; *u* is a constant (fixed) parameter replaced below by a control $u(t)$. By $\varphi(x,u,t)$ we denote its solution, with $\varphi(x,u,0)=x$. By L_fh we denote the Lie derivative of h along f and we denote $L_f^sh=L_f^{s-1}(L_fh),s\ge1;\ L_f^0h=h.$ Let $H(x)=(L_fh(x),s=0,1,...,n-1).$ Let ρ be a relative degree of $(\sum_n,h)$, i.e. the minimal integer *s* for which $\frac{\partial^sh(\varphi(x,u,t))}{\partial t^s}\Big|t=0$ depends on *u* explicitly. Consider the system of algebraic equations $H(z)=w$ in the complex space, i.e. $z,w\in\mathbf{C}^n$. Suppose that *H* satisfies the following condition called below the invertibility condition: there is a *n*-variate polynomial $\alpha(w)$ for which the system $H(z)=w_*$ has a unique solution for any w_* such that $\alpha(w_*)\ne0$. Now we establish

Proposition 1. *Assume that* $\rho=n$. *Then the mapping* H *maps* $\sum_n$ *into the system* $\sum_n':\dot{y}_1=y_2;...,\dot{y}_{n-1}=y_n;\dot{y}_n=A(y)+uB(y):=p_1(y)q_1^{-1}(y)+up_2(y)q_2^{-1}(y),y\in\mathbf{R}^n,$ *for some real polynomials* $p_s;q_s,s=1,2$.

Proof. By applying of Proposition 3.3, (Ploski, 1985), to complexified polynomials $L_f^sh,s=0,1,...n-1,$ $L_gL_f^{n-1}h$ in the complex space and coming back to the real space we deduce that there are real polynomials $p_s;q_s,s=1,2$, of *n* real variables such that $L_f^nh=p_1(H)q_1^{-1}(H);L_gL_f^{n-1}h=p_2(H)q_2^{-1}(H)$. These identities entail the identity $L_f^nh+uL_gL_f^{n-1}h=p_1(H)q_1^{-1}(H)+up_2(H)q_2^{-1}(H).$ Thus the first condition implies the desirable result. At last, we note that if each of pairs of polynomials $p_s, q_s,$ is uncancellable, $s = 1,2,$ then $\sum'_n$ constructed by this method is defined on the largest open set in $\mathbf{R}^n$.

This result gives only sufficient conditions for obtaining $\sum'_n$. Below we shall use "almost" global elimination of *x* in order to construct $\sum'_n$.

Proposition 2. *Let* $\rho = n$. *We introduce a polynomial* $w(y) := p_2(y)q_1(y)q_2(y)$. *Assume that there is an equilibrium point* x_* *of the system* $\sum_n$ *with u be fixed, such that* $y_* := H(x_*) \in C(w),$ *with* $C(w)$ *be a connected component of the set* $\mathbf{R}^n - w^{-1}(0)$. *If* U *is a domain in* $H^{-1}(C(w)),$ *such that* $x_* \in U,$ *then there is a controller*

$$u(t) := u + \delta u(t, x(t)) \tag{1}$$

such that $\sum_n$ *exhibits chaos on* U *provided the parameter* u *is replaced by* $u(t)$ *defined in* (1).

Proof. By using the change of coordinates $z = y - y_*$ we obtain a system denoted as $\sum''_n$ with $z = 0$ as an equilibrium point. We introduce the set $C^{sh}(w) := \{z \in \mathbf{R}^n | z + y_* \in C(w)\}$ and by U_1 we denote any open connected subset in $C^{sh}(w)$ such that $z = 0$ is contained in U_1 and $H(U) \subset U_1$. The set $w^{-1}(0)$ will be called singular. We form the equation for finding the controller δu creating chaos for $\sum''_n$ in U_1: $p_1(z(t) + y_*)q_1^{-1}(z(t) + y_*) + (u + \delta u)p_2(z(t) + y_*)q_2^{-1}(z(t) + y_*) + \sum_{s=0}^{n-1} \gamma_s z_{s+1}(t) = v = \varepsilon \sin(\xi$ $z_1(t - \tau))$ for any Hurwitz stable polynomial $\lambda^n + \sum_{s=0}^{n-1} \gamma_s \lambda^s, \gamma_0 \neq 0.$ The last formula and arguments in (Wang et al.,2000, p.775) related to exact linearization approach entail the expression for the controller for $\sum''_n : \delta u = (v(t) - p_1 q_1^{-1} - \sum_{s=0}^{n-1} \gamma_s z_{s+1}) p_2^{-1} q_2 - u,$ with omitted arguments for $p_s ; q_s, s = 1,2$. So by (Wang et al., 2000, p.773) we conclude that $\sum''_n$ is chaotic in U_1 for sufficiently large value of the delay τ. The restriction $H|U$ is a diffeomorphism on its image. Therefore coming back to x-coordinates, we receive the final formula for the time-delay feedback:

$$\delta u = (\varepsilon \sin(\xi(h(x(t-\tau)) - h(x_*))) - p_1(H(x(t))) q_1^{-1}(H(x(t))) - \sum_{s=0}^{n-1} \gamma_s (L_f^s h(x(t)) - L_f^s h(x_*))) p_2^{-1}(H(x(t))) q_2(H(x(t))) - u \tag{2}$$

which makes the system $\sum_n$ chaotic in U with $x_* \in U.$

Arguing like in (Wang et al., 2000), we can take v as a controller, while δu is considered as a coordinate transformation (2).

4 Application 1: The Lorenz system with the control parameter b

Let us take the uncontrolled Lorenz system $\sum_3^L: \dot{x}_1 = -\sigma x_1 + \sigma x_2; \dot{x}_2 = rx_1 - x_2 - x_1 x_3; \dot{x}_3 = -bx_3 + x_1 x_2,$ written as $\dot{x} = f(x)$. In this section we study the possibility to chaotify the Lorenz system by using *b* as a control parameter; here $\sigma, b > 0; r > 1$. We remind here that in (Starkov and Chen, 2004) the chaotification of the Lorenz system by using *r* as a control parameter was examined. Let us choose $h(x) = x_1$ and we get

$$
\begin{aligned}
L_f h(x) &= \sigma(x_2 - x_1) \\
L_f^2 h(x) &= -\sigma x_2 + \sigma^2 x_1 - \sigma x_2 - \sigma x_1 x_3 + \sigma r x_1 \\
L_f^3 h &= x_1(-\sigma^3 - 2\sigma^2 r - \sigma r) + x_2(\sigma^3 + \sigma^2 r + \sigma^2 \\
&+ \sigma) - \sigma^2 x_3 x_2 + (2\sigma^2 + \sigma + \sigma b) x_1 x_3 - \sigma x_1^2 x_2
\end{aligned}
$$

Therefore

$$
\begin{aligned}
x_1 &= h \\
x_2 &= \sigma^{-1} L_f h + h \\
x_3 &= \frac{-L_f^2 h + h(\sigma r - \sigma) - L_f h(\sigma + 1)}{\sigma h}
\end{aligned}
$$

and it is clear that Proposition 1 is applicable. By using y-coordinates, $y_1 = h(x); y_2 = L_f h(x); y_2 = L_f^2 h(x)$ we get the normal form

$$
\begin{aligned}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= y_3 \\
\dot{y}_3 &= y_3(-1 - b - \sigma) + y_2(-b - b\sigma) + y_1(-b\sigma + b\sigma r) + \frac{1}{y_1} y_3 y_2 - \sigma y_1^3 + \frac{\sigma}{y_1} y_2^2 + \frac{1}{y_1} y_2^2 - y_1^2 y_2
\end{aligned}
$$

The equation of the controller is given by the formula

$$
\begin{aligned}
&p_{11}(y) + p_{12}(y) q_1^{-1}(y) + (b + \delta b) p_2(y) + \\
&\sum_{s=0}^{2} \gamma_s (y_{s+1} - y_{s+1^*}) =
\end{aligned}
\quad = v := \varepsilon \sin(\xi(y_1(t - \tau) - \sqrt{b(r-1)})),
$$

With

$$
\begin{aligned}
p_{11}(y) &= -y_3(\sigma + 1) - \sigma y_1^3 - y_1^2 y_2 \\
p_{12}(y) &= y_3 y_2 + (\sigma + 1) y_2^2
\end{aligned}
$$

$$q_1(y) = y_1$$
$$p_2(y) = -y_3 - y_2(\sigma+1) + y_1(\sigma r - \sigma)$$

So the equation for the anticontroller is

$$\delta b = (v - \sum_{s=0}^{2} \gamma_s (L_f^s h(x) - L_f^s h(x_*)) - p_{11}(H(x)) -$$
$$p_{12}(H(x)) q_1^{-1}(H(x))) p_2^{-1}(H(x)) - b$$

In x-coordinates we have

$$p_{11}(H(x)) = x_1(-\sigma^2 - \sigma^3 - \sigma r - \sigma^2 r) + x_2(2\sigma^2 + \sigma^3 + \sigma) + x_1 x_3(\sigma^2 + \sigma) - \sigma x_1^2 x_2$$
$$p_{12}(H(x)) = x_1{}^2(-\sigma^2 r + \sigma^2) + x_1 x_2(\sigma^2 r - \sigma^2) -$$
$$\sigma^2 x_1 x_2 x_3 + \sigma^2 x_1^2 x_3$$
$$q_1(H(x)) = x_1$$
$$p_2(H(x)) = \sigma x_1 x_3$$

Using the formula for $A(y) + bB(y)$ and (2) and collecting terms with b we compute the singular set. It is described by equations $y_1 = 0;\ y_1(\sigma r - \sigma) - y_2(1+\sigma) = y_3$ or in x-coordinates by

$$-(-\sigma x_1 + \sigma x_2)(1+\sigma) - x_1(\sigma^2 + \sigma r) - x_2(-\sigma^2 - \sigma) + \sigma x_1 x_3 + \sigma r x_1 - \sigma x_1 = \sigma x_1 x_3 = 0$$

Thus the Lorenz system restricted on any open domain $D \ni Z_+$ inside the quadrant $x_1 > 0; x_3 > 0,$ exhibits chaos for the enhanced area of parameters provided the time-delay feedback $b(t) = b + \delta b(t, x(t))$ (b is replaced by $b(t)$ in $\sum_3^L$) is used for sufficiently large $\tau > 0$.

Let us choose $\gamma_0 = 6, \gamma_1 = 11$ and $\gamma_2 = 6$.We take $\tau = 1$ with $a = 1, b = 10$ and $r = 28$.

Fig. 1 shows the chaotic attractor generated by the linear system

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = y_3$$
$$\dot{y}_3 = \varepsilon \sin(\xi(y_1(t-\tau) - \sqrt{b(r-1)})) -$$
$$6(y_1 - \sqrt{b(r-1)}) - 11y_2 - 6y_3,$$

where $\varepsilon = 1, \xi = 63$. Fig. 2 contains the chaotic attractor of the Lorenz system using chaotification, with b as the control parameter and the initial point $(y_1(0), y_2(0), y_3(0)) = (16.43, 0, 0)$

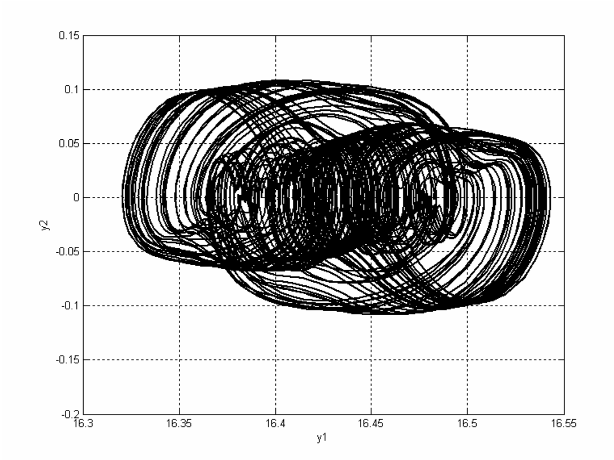

Fig. 1. The chaotic attractor of the linear system.

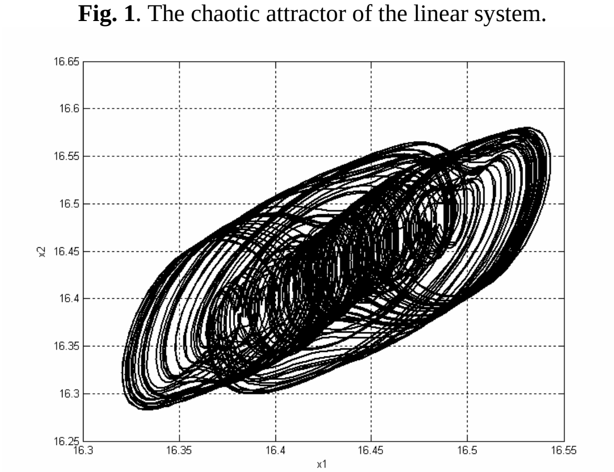

Fig. 2. The chaotic attractor of the Lorenz system, with b as the control parameter.

5 Application 2: The Burke-Shaw system with the control parameter κ

In this section we take the Burke-Shaw system which was described in (Shaw, 1981) in the context of the qualitative study of the Lorenz attractor

$$\dot{x}_1 = -s(x_1 + x_2)$$
$$\dot{x}_2 = -x_2 - sx_1x_3$$
$$\dot{x}_3 = \kappa + sx_1x_2$$

We choose $h(x) = x_1$ and compute

$$L_f h(x) = -s(x_1 + x_2)$$
$$L_f^2 h(x) = s^2 x_1 + x_2(s^2 + s) + s^2 x_1 x_3$$
$$L_f^3 h = x_1(-s^3 + s^2\kappa) + x_2(-s^3 - s^2 - s) - 2s^3 x_1 x_3 - s^3 x_2 x_3 - s^2 x_1 x_3 + s^3 x_1^2 x_2$$

and it is clear that Proposition 1 is applicable. By solving this system of equations we obtain that

$$x_1 = h$$
$$x_2 = \frac{L_f h + sh}{-s}$$
$$x_3 = \frac{L_f^2 h - s^2 h - s^2 (\frac{L_f h + sh}{-s}) - s(\frac{L_f h + sh}{-s})}{s^2 h}$$

In y -coordinates $y_1 = h(x);$ $y_2 = L_f h(x);$
$y_2 = L_f^2 h(x)$ we get the normal form

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = y_3$$
$$\dot{y}_3 = s^2 \kappa y_1 - y_3(s+1) + \frac{1}{y_1}(y_2 y_3 + s y_2^2 + y_2^2) - s^2 y_1^2 y_2 - s^3 y_1^3$$

The equation of the controller has the form

$$\begin{aligned} & p_{11}(y) + p_{12}(y) q_1^{-1}(y) + (\kappa + \delta\kappa) p_2(y) + \\ & \sum_{s=0}^{2} \gamma_s (y_{s+1} - y_{s+1*}) = \end{aligned} = v := \varepsilon \sin(\xi(y_1(t-\tau) - \sqrt{\frac{\kappa}{s}})),$$

with

$$p_{11}(y) = -y_3(s+1) - s^2 y_1^2 y_2 - s^3 y_1^3$$
$$p_{12}(y) = y_2 y_3 + s y_2^2 + y_2^2$$
$$q_1(y) = y_1$$
$$p_2(y) = s^2 y_1$$

So the equation for the anticontroller is

$$\delta\kappa = (v - \sum_{s=0}^{2} \gamma_s (L_f^s h(x) - L_f^s h(x_*)) - p_{11}(H(x)) -$$
$$p_{12}(H(x)) q_1^{-1}(H(x))) p_2^{-1}(H(x)) - \kappa$$

In x -coordinates we have

$$p_{11}(H(x)) = -x_1(s^3 + s^2) - x_2(s^3 + 2s^2 + s) - - x_1 x_3 (s^3 + s^2) + s^3 x_1^2 x_2$$
$$p_{12}(H(x)) = s^2 x_1{}^2 + s^2 x_1 x_2 - s^3 x_1^2 x_3 - s^3 x_1 x_2 x_3$$
$$q_1(H(x)) = x_1$$
$$p_2(H(x)) = s^2 x_1$$

Using the formula for $A(y)+\kappa B(y)$ and (2) and collecting terms with κ we compute the singular set. It is described by equations $y_1 = 0;\ s^2 y_1 = 0$ or in x-coordinates by

$$x_1 = 0; s^2 x_1 = 0 .$$

Thus the Burke-Shaw system restricted on any open domain $D \ni Z_+$ inside the semispace $x_1 > 0$ or the semispace $x_1 < 0$, exhibits chaos for the enhanced area of parameters provided the time-delay feedback $\kappa(t) = \kappa + \delta\kappa(t, x(t))$ (κ is replaced by $\kappa(t)$ in $\sum_3^L$) is used for sufficiently large $\tau > 0$.

Let us choose $\gamma_0 = 1, \gamma_1 = 3, \gamma_2 = 3, \kappa = 25$ and $s = 1$.We take $\tau = 1$. Fig. 3 shows the chaotic attractor generated by the linear system

$$\begin{gathered}\dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = \varepsilon \sin(\xi(y_1(t-\tau) - \sqrt{\frac{\kappa}{s}})) - (y_1 - \sqrt{\frac{\kappa}{s}}) \\ -3y_2 - 3y_3\end{gathered}$$

where $\varepsilon = 1, \xi = 68$. Fig. 4 contains the chaotic attractor of the Burke- Shaw system using chaotification, with κ as the control parameter and the initial point $(y_1(0), y_2(0), y_3(0)) = (5.01, 0.01, 0.01)$.

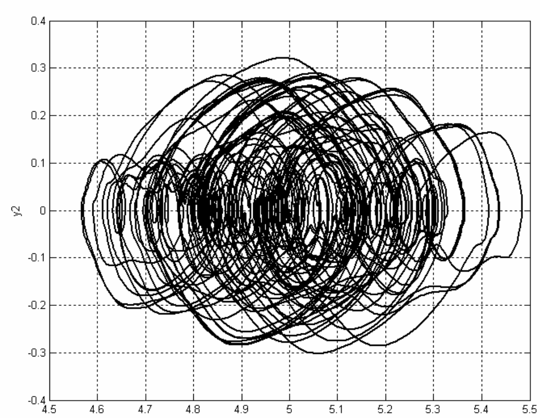

Fig. 3. The chaotic attractor of the linear system.

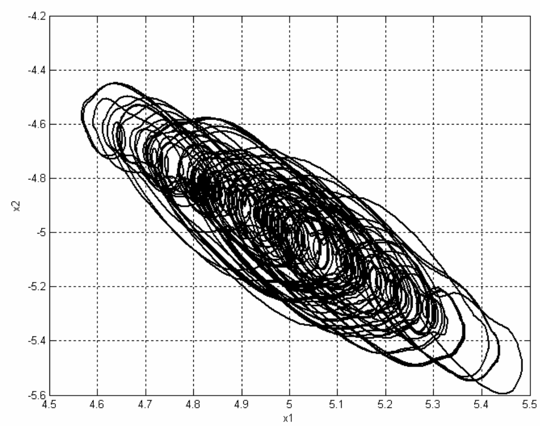

Fig. 4. The chaotic attractor of the Burke- Shaw system, with κ as the control parameter.

6 Application 3: The Liu system with the control parameter b

Very recently, the new chaotical system was proposed, see in (Liu et al.,2004). We denote it by

$$\dot{x}_1 = a(x_2 - x_1)$$
$$\dot{x}_2 = bx_1 - kx_1x_3$$
$$\dot{x}_3 = -cx_3 + \theta x_1^2$$

We choose $h(x) = x_1$ and compute

$$L_f h(x) = -a(x_2 - x_1)$$
$$L_f^2 h(x) = -a^2 x_2 + a^2 x_1 + abx_1 - akx_1x_3$$
$$L_f^3 h = x_1(-a^3 - 2a^2 b) + x_2(a^3 + a^2 b) + (2a^2 k + akc)x_1x_3 - a^2 kx_2x_3 - ak\theta x_1^3$$

Solving these equations respecting x we get

$$x_1 = h$$
$$x_2 = a^{-1} L_f h + h$$
$$x_3 = \frac{-L_f^2 h - aL_f h + abh}{akh}$$

and it is clear that Proposition 1 is applicable. In y -coordinates $y_1 = h(x); y_2 = L_f h(x); y_2 = L_f^2 h(x)$ we get the normal form

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = y_3$$

$$\dot{y}_3 = \frac{1}{y_1} y_2 y_3 - a y_3 + \frac{a}{y_1} y_2^2 - c y_3 - ac y_2 + abc y_1 - ak\theta y_1^3$$

The equation of the controller

$$\begin{aligned} & p_{11}(y) + p_{12}(y) q_1^{-1}(y) + (b + \delta b) p_2(y) + \\ & \sum_{s=0}^{2} \gamma_s (y_{s+1} - y_{s+1^*}) = \end{aligned} = v := \varepsilon \sin(\xi(y_1(t-\tau) - \sqrt{\frac{cb}{\theta k}})),$$

with

$$\begin{gathered} p_{11}(y) = -y_3(a+c) - acy_2 - ak\theta y_1^3 \\ p_{12}(y) = y_2 y_3 + y_2^2 \\ q_1(y) = y_1 \\ p_2(y) = acy_1 \end{gathered}$$

So the equation for the anticontroller is

$$\begin{aligned} & \delta b = (v - \sum_{s=0}^{2} \gamma_s (L_f^s h(x) - L_f^s h(x_*)) - \\ & p_{11}(H(x)) - p_{12}(H(x)) q_1^{-1}(H(x))) \\ & p_2^{-1}(H(x)) - b \end{aligned}$$

In x-coordinates we have

$$\begin{gathered} p_{11}(H(x)) = a^3 x_2 - a^3 x_1 - a^2 b x_1 + a^2 k x_1 x_3 - abc x_1 + ack x_1 x_3 - ak\theta x_1^3 \\ p_{12}(H(x)) = -a^3 x_2^2 + 2a^3 x_1 x_2 + a^2 b x_1 x_2 - \\ a^2 k x_1 x_2 x_3 - a^3 x_1^2 - a^2 b x_1^2 + a^2 k x_1^2 x_3 + a^2 x_2^2 - \\ 2a^2 x_1 x_2 + a^2 x_1^2 \\ q_1(H(x)) = x_1 \\ p_2(H(x)) = acx_1 \end{gathered}$$

Using the formula for $A(y) + bB(y)$ and (2) and collecting terms with b we compute the singular set. It is described by equations $y_1 = 0; acy_1 = 0$ or in x-coordinates by

$$x_1 = 0; acx_1 = 0$$

Thus the Liu system restricted on any open domain $D \ni Z_+$ inside the semispace $x_1 > 0$ or the semispace $x_1 < 0$ exhibits chaos for the enhanced area of parameters provided the time-delay feedback $b(t) = b + \delta b(t, x(t))$ (b is replaced by $b(t)$ in $\sum_3^L$) is used for sufficiently large $\tau > 0$.

Let us choose $\gamma_0 = 6$, $\gamma_2 = 11$, and $\gamma_1 = 6$.We take $\tau = 1$ with $a = 1$, $b = 40$, $k = 1$, $c = 2.5$ and $\theta = 4$. Fig. 5 shows the chaotic attractor generated by the linear system

$$\begin{gathered}\dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = \varepsilon \sin(\xi(y_1(t-\tau) - \sqrt{\frac{cb}{\theta k}})) - 6(y_1 - \sqrt{\frac{cb}{\theta k}}) \\ -11y_2 - 6y_3\end{gathered}$$

where $\varepsilon = 1, \xi = 30$. Fig. 6 contains the chaotic attractor of the Liu system using chaotification, with b as the control parameter and the initial point $(y_1(0), y_2(0), y_3(0)) = (5.025, 0.001, 0.001)$

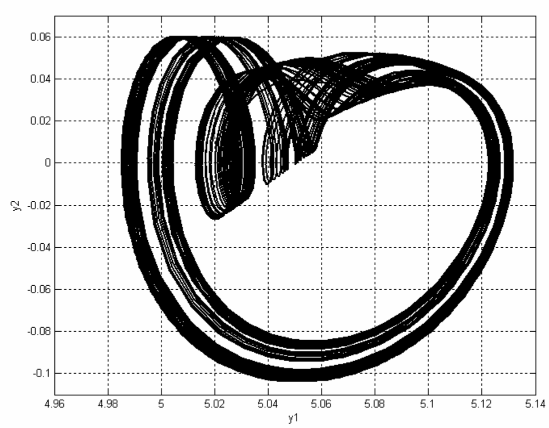

Fig. 5. The chaotic attractor of the linear system

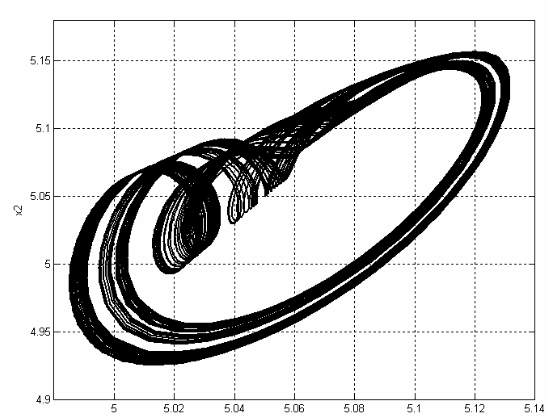

Fig. 6. The chaotic attractor of the Liu system, with b as the control parameter.

7 Conclusions

Semiglobal chaotification problem of one class of polynomial continuous- time systems is studied. Its solution is based on transforming of a system of this type into a *n*-th order explicit scalar differential equation with a rational nonlinearity and application of a time-delay feedback proposed by Wang, Chen and Yu. It is shown that our approach is efficient to different three-dimensional systems with a quadratic right- side. As examples, the Lorenz system, the Burke- Shaw system and the Liu system are considered. Our future researching challenge is to apply these ideas to some class of hybrid polynomial systems (polynomial systems having discontinuities of a special type).

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