RESUMEN DE TESIS DOCTORAL

Alpha-Beta Bidirectional Associative Memories

Memorias Asociativas Bidireccionales alfa-beta

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Abstract

Most models of Bidirectional associative memories intend to achieve that all trained pattern correspond to stable states; however, this has not been possible. Also, none of the former models has been able to recall all the trained patterns. In this work we introduce a new model of bidirectional associative memory which is not iterative and has no stability problems. It is based on the Alpha-Beta associative memories. This model allows perfect recall of all trained patterns, with no ambiguity and no conditions. Applications of Alpha-Beta Bidirectional Associative Memories as fingerprint recognition and translator are presented.
Keywords: Bidirectional Associative Memories, Alpha-Beta Associative Memories, correct recall.

Resumen

Los modelos de memorias asociativas bidireccionales, en su mayoría, pretenden lograr que todos los patrones entrenados correspondan a estados estables; sin embargo, esto no ha sido posible. Además, ninguno de esos modelos ha podido recuperar todos los patrones entrenados. En este trabajo se presenta un nuevo modelo de memoria asociativa bidireccional que no es iterativo y no tiene problemas de estabilidad; está basado en las memorias asociativas Alfa-Beta. Este modelo permite la recuperación perfecta de todos los patrones entrenados sin ambigüedades y sin condiciones. Se presentan dos ejemplos de aplicación: el primero de reconocimiento de huellas digitales, y el segundo es un traductor.
Palabras clave: Memorias asociativas bidireccionales, memorias asociativas Alfa-Beta, recuperación correcta.

1 Introduction

The first bidirectional associative memory (BAM), introduced by Kosko (Kosko, 1988), was the base of many models presented later. Some of this models substituted the learning rule for an exponential rule (Jeng, 1990; Wang, 1992; Chen, 1997); others used the method of multiple training and dummy addition in order to reach a greater number of stable states (Wang, 1990), trying to eliminate spurious states. With the same purpose, linear programming techniques (Wang, 1991), the descending gradient method (Perfetti, 1993; Zheng, 2005) have been used, besides genetic algorithms (Shen, 2005) and BAM with delays (Arik, 2005; Park, 2006). Other models of non iterative bidirectional associative memories exist, such as morphological BAM (Ritter, 1999) and Feedforward BAM (Wu, 2000). All these models have arisen to solve the problem of low pattern recall capacity shown by the BAM of

Kosko; however, none has been able to recall all the trained patterns. Also, these models demand the fulfillment of some specific conditions, such as a certain Hamming distance between patterns, solvability by linear programming, orthogonality between patterns, among other.

The model of bidirectional associative memory presented in this paper is based on the Alpha-Beta associative memories (Yáñez, 2002), is not an iterative process, and does not present stability problems. Pattern recall capacity of the Alpha-Beta BAM is maximal, being $2^{\min(n,m)}$, where m and n are the input and output patterns dimension, respectively. Also, it always shows perfect pattern recall without imposing any condition.

In section 2 we present the Alpha-Beta autoassociative memories, base of our new model of BAM, and the theoretical sustentation of Alpha-Beta BAM. In section 3 the model is applied to fingerprint recognition and translator. Conclusions follow in section 4.

2 Alpha-Beta Bidirectional Associative Memories

In this section the proposed model of bidirectional associative memory is presented. However, since it is based on the Alpha-Beta autoassociative memories, a summary of this model will be given before presenting our proposal, the new model of bidirectional associative memory.

2.1 Alpha-Beta Associative Memories

Basic concepts about associative memories were established three decades ago in (Kohonen, 1972; Kohonen, 1989; Hassoun, 1993), nonetheless here we use the concepts, results and notation introduced in the Yáñez-Márquez's PhD Thesis (Yáñez, 2002). An associative memory **M** is a system that relates input patterns, and outputs patterns, as follows: $\mathbf{x}\rightarrow\mathbf{M}\rightarrow\mathbf{y}$ with **x** and **y** the input and output pattern vectors, respectively. Each input vector forms an association with a corresponding output vector. For k integer and positive, the corresponding association will be denoted as $\left(x^k, y^k\right)$.

If μ is an index, the fundamental set is represented as: $\left\{\left(x^\mu, y^\mu\right) \mid \mu = 1,2,\ldots,p\right\}$ with p the cardinality of the set. The patterns that form the fundamental set are called fundamental patterns. If it holds that $x^\mu = y^\mu, \forall \mu \in \{1,2,\ldots,p\}$, **M** is *autoassociative*, otherwise it is *heteroassociative*; in this case it is possible to establish that $\exists \mu \in \{1,2,\ldots,p\}$ for which $x^\mu \neq y^\mu$. A distorted version of a pattern x^k to be recuperated will be denoted as $\tilde{x}^k$. If when feeding a distorted version of x^ϖ with $\varpi = \{1,2,\ldots,p\}$ to an associative memory **M**, it happens that the output corresponds exactly to the associated pattern y^ϖ, we say that recuperation is perfect.

Among the variety of associative memory models described in the scientific literature, there are two models that, because of their relevance, it is important to emphasize: morphological associative memories which were introduced by Ritter *et. al.* (Ritter, 1998), and Alpha-Beta associative memories, which were introduced in (Yáñez, 2002; Yáñez, 2003; Yáñez, 1999).

The Alpha-Beta associative memories are of two kinds and are able to operate in two different modes. The operator α is useful at the learning phase, and the operator β is the basis for the pattern recall phase. Let the sets A and B be, A={0,1} and B={0,1,2}, then the operators α and β are defined in tabular form:

$\alpha : A \times A \rightarrow B$

x	y	$\alpha(x,y)$
0	0	1
0	1	0
1	0	2
1	1	1

$\beta : B \times A \rightarrow A$

x	y	$\beta(x,y)$
0	0	0
0	1	0
1	0	0
1	1	1
2	0	1
2	1	1

The sets A and B, the α and β operators, along with the usual $\wedge$ (minimum) y $\vee$ (maximum) operators, form the algebraic system $(A, B, \alpha, \beta, \wedge, \vee)$ which is the mathematical basis for the Alpha-Beta associative memories. An Alpha-Beta autoassociative memory has the fundamental set:

$$\{(\mathbf{x}^\mu, \mathbf{x}^\mu) \mid \mu = 1, 2, ..., p\} \quad \text{where } \mathbf{x}^\mu \in A^n$$

Alpha-Beta autoassociative memories *max* and *min* (**V** y **Λ**) are calculated in learning phase by following equations:

$$\mathbf{V} = \bigvee_{\mu=1}^{p} \left[\mathbf{x}^\mu \otimes (\mathbf{x}^\mu)^t\right] \quad \text{and} \quad \mathbf{\Lambda} = \bigwedge_{\mu=1}^{p} \left[\mathbf{x}^\mu \otimes (\mathbf{x}^\mu)^t\right]$$

donde las *ij*-ésimas componentes de ambas matrices son:

$$v_{ij} = \bigvee_{\mu=1}^{p} \alpha\left(x_i^\mu, x_j^\mu\right) \quad \text{and} \quad \lambda_{ij} = \bigwedge_{\mu=1}^{p} \alpha\left(x_i^\mu, x_j^\mu\right)$$

In recall phase, when a pattern $\mathbf{x}^\mu$ is presented to memories **V** and Λ, the *i*-th components of recalled patterns are:

$$\left(\mathbf{V} \Delta_\beta \mathbf{x}^\omega\right)_i = \bigwedge_{j=1}^{n} \beta(v_{ij}, x_j^\omega) \text{ and } \left(\mathbf{\Lambda} \nabla_\beta \mathbf{x}^\omega\right)_i = \bigvee_{j=1}^{n} \beta(\lambda_{ij}, x_j^\omega)$$

2.2 Alpha-Beta Bidirectional Associative Memories

The model proposed in this paper has been named Alpha-Beta BAM since Alpha-Beta autoassociative memories, both *max* and *min*, play a central role in the model design. However, before going into detail over the processing of an Alpha-Beta BAM, we will define the following.

Definition 1 (One-Hot) *Let the set A be* $A = \{0, 1\}$ *and* $p \in \mathbf{Z}^+$, $p > 1$, $k \in \mathbf{Z}^+$, *such that* $1 \leq k \leq p$. *The k-th one-hot vector of p bits is defined as vector* $h^k \in A^p$ *for which it holds that the k-th component is* $h_k^k = 1$ *and the ret of the components are* $h_j^k = 0$, $\forall j \neq k$, $1 \leq j \leq p$.

Definition 2 (Zero-Hot) *Let the set A be* $A = \{0, 1\}$ *and* $p \in \mathbf{Z}^+$, $p > 1$, $k \in \mathbf{Z}^+$, *such that* $1 \leq k \leq p$. *The k-th zero-hot vector of p bits is defined as vector* $\overline{\mathbf{h}}^k \in A^p$ *for which it holds that the k-th component is* $h_k^k = 0$ *and the ret of the components are* $h_j^k = 1$, $\forall j \neq k$, $1 \leq j \leq p$.

Definition 3 (Expansion vectorial transform) *Let the set A be* $A = \{0, 1\}$ *and* $n \in \mathbf{Z}^+$,*y* $m \in \mathbf{Z}^+$. *Given two arbitrary vectors* $\mathbf{x} \in A^n$ *and* $\mathbf{e} \in A^m$, *the expansion vectorial transform of order m,* $\tau^e : A^n \rightarrow A^{n+m}$, *is defined as* $\tau^e(\mathbf{x}, \mathbf{e}) = \mathbf{X} \in A^{n+m}$, *a vector whose components are:* $X_i = x_i$ *for* $1 \leq i \leq n$ *and* $X_i = e_i$ *for* $n + 1 \leq i \leq n + m$.

Definition 4 (Contraction vectorial transform) *Let the set A be $A = \{0, 1\}$ and $n \in \mathbf{Z}^+$, y $m \in \mathbf{Z}^+$ such that $1 \leq m < n$. Given one arbitrary vector $\mathbf{X} \in A^{n+m}$, the contraction vectorial transform of order m, $\tau^c : A^{n+m} \rightarrow A^m$, is defined as $\tau^c(\mathbf{X}, m) = \mathbf{c} \in A^m$, a vector whose components are: $c_i = X_{i+n}$ for $1 \leq i < m$.*

In both directions, the model is made up by two stages, as shown in figure 1.

For simplicity, it will be described the necessary process in one direction (from **x** to **y**), the process in the complementary direction (from **y** to **x**) is similar to first, **y** for **x** are replace and viceversa.

Stage 1 is the main proposition of this model and his task is: given a $\mathbf{x}^k$ or a noisy version of it ($\tilde{\mathbf{x}}^k$), the *one-hot* vector $\mathbf{h}^k$ must be obtained without ambiguity and with no condition (see Figure 2). Now, as an input to Stage 2 we have one element of a set of p orthonormal vectors. Stage 2 is a variation of the *Linear Associator* and remember that the *Linear Associator* has perfect recall when it works with orthonormal vectors. Thus, the task of Stage 2 is to obtain $\mathbf{y}^k$, parting from a *one-hot* vector $\mathbf{h}^k$.

Steps for construction for Stages 1 and 2 in learning phase can be observed in figure 3. Expansion vectorial transform is applied to every input pattern $\mathbf{x}^k$ with their respective *one-hot* and *zero-hot* vectors to, subsequently, construct both *max* and *min* Alpha-Beta autoassociative memories. For the construction of the modified *Linear Associator*, each column in the matrix $\mathbf{LA}_y$ corresponds to each output pattern $\mathbf{y}^k$.

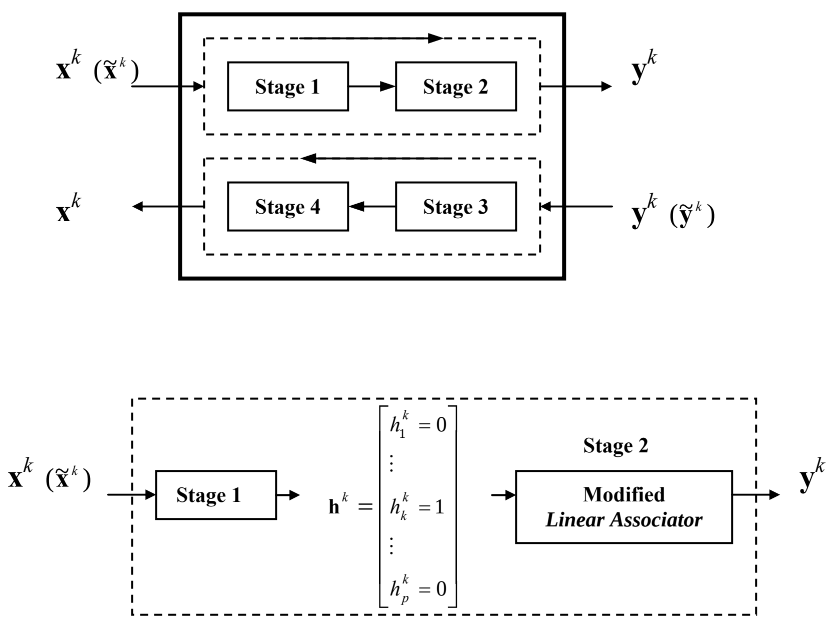


Fig. 1. Graphical schematics of the Alpha-Beta bidirectional associative memory

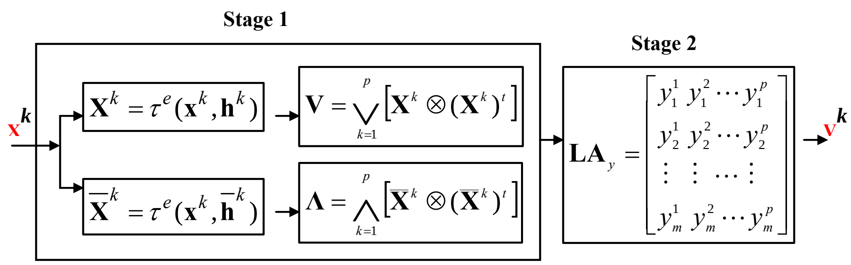


Fig. 2. Schematics of the process done in the direction from **x** to **y**. Here are shown only Stage 1 and Stage 2.

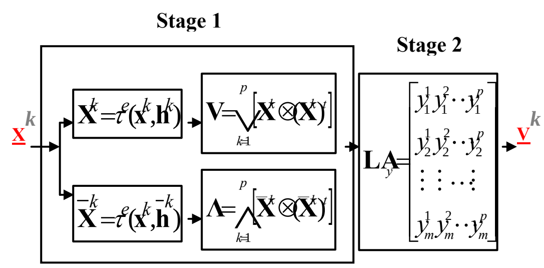


Fig. 3. Construction of Stage 1 and 2 in learning phase for Alpha-Beta BAM.

Recall phase is described through the chart showed in figure 4.

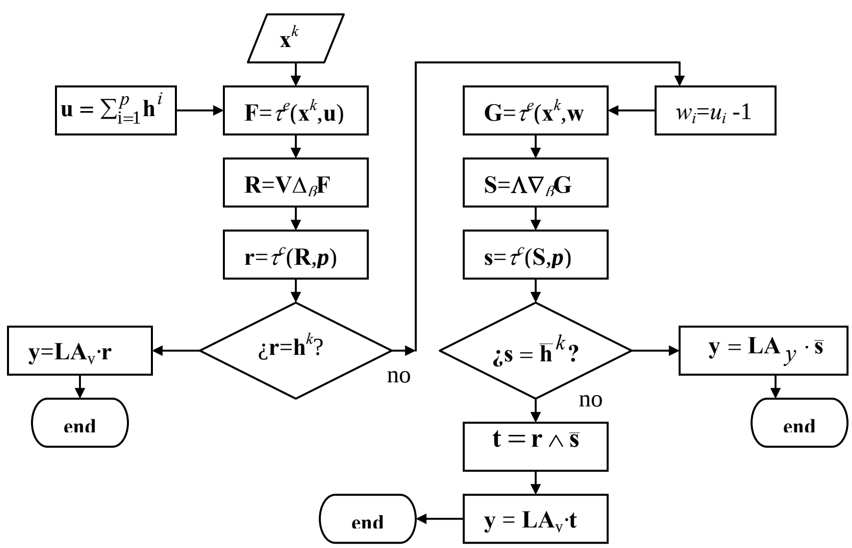


Fig. 4. Steps for recall phase in $\mathbf{x} \rightarrow \mathbf{y}$ direction for Alpha-Beta Bidirectional Associative Memory.

3 Simulations

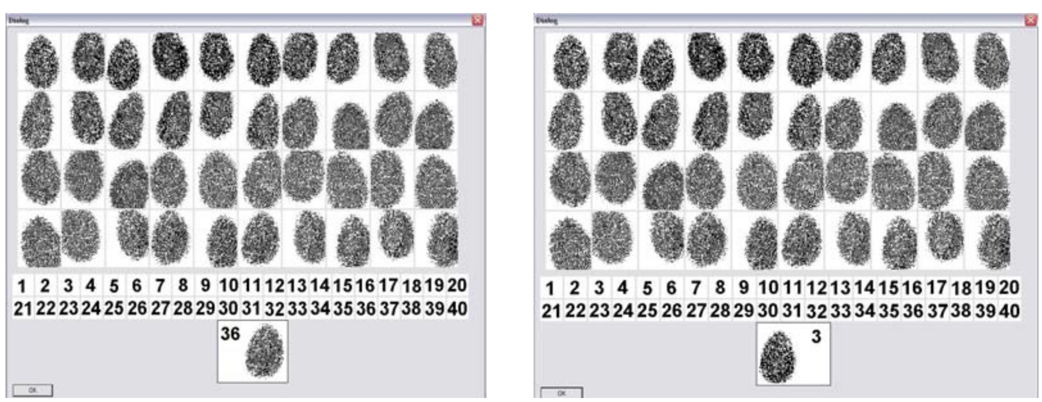
In this section a fingerprint identifier and an English/Spanish-Spanish/English translator are presented as applications of Alpha-Beta Bidirectional Associative Memories.
The programming language used to implement the code of both applications is Visual C++ 6.0.

3.1 Fingerprint identifier

The Alpha-Beta BAM model was applied as a Fingerprint Identifier. The fingerprints used were obtained from the Fingerprint Verification Competition (FPV2000) located at the web page http://bias.csr.unibo.it/fvc2000/download.asp. Originally the images have dimensions of 240 x 320 pixels. The image editing software *Advanced Batch Converter* was used to crop the size of the images in order to fit 40 fingerprints in one screen, thus ending with dimensions of 80 x 170 pixels. Each fingerprint was associated to a number, from 1 to 40 inclusive. These numbers are images of 10 x 10 pixels. Both sets of images, fingerprints and numbers, are binary.

The process of creation of the Alpha-Beta BAM amounts to 1 minute and 34 seconds, approximately. For this purpose a Sony® VAIO® laptop, equipped with an Intel® h® 4 processor at 2.8 GHz was used.
The program allows choosing an input pattern among both sets of images. If the chosen input pattern is a number then, automatically, the chosen number and its corresponding output pattern, which will be a fingerprint, are shown (see figure 5a). If the chosen image is a fingerprint, then the fingerprint and its corresponding recalled pattern are shown (see figure 5b).

First, we choose as input patterns each one of the 40 fingerprints, and all corresponding output patterns were perfectly recalled. Then, every number image was chosen as input pattern, and again the corresponding fingerprints were perfectly recalled.



(a) (b)

Fig. 5. The bidirectional process of the model is shown in both screens. (a) A number is chosen as input pattern and its corresponding fingerprint is perfectly recalled. (b) The input pattern is now a fingerprint, its corresponding output pattern a number.

3.2 English/Spanish-Spanish/English translator

This software has the ability to translate words from English to Spanish and visceversa. For learning phase two text files were used, containing 120 words in English and Spanish, respectively. With these two files, the Alpha-Beta Bidirectional Associative Memory is built (see figure 6).

The learning phase lasts, 1 minute and 6 seconds, when the program was run on a 2.8 GHz Pentium 4 Sony VAIO Laptop.

Once Alpha-Beta BAM has been built, a word is written during the recalling phase, aither in English or Spanish, and the translation mode is selected. Immediately the word appears in the corresponding language. A example of this can be seen in figure 7, where the word to be translated was "accuracy" and its corresponding translation to Spanish is "exactitud".

Fig. 6. The Alpha-Beta Bidirectional Associative Memory is built by associating 120 words in Spanish with 120 words in English, which are contained in two text files.

The translator offers other advantages as web. For instance, suppose only part of the word is entered, say "accur" instead of "accuracy"; the program will give as result "exactitud" (see figure 8).

Fig. 7. The word to be translated is written –in this example it was "accuracy"- and translation mode is selected; immediately, its corresponding Spanish word appears, which in this case is "exactitud".

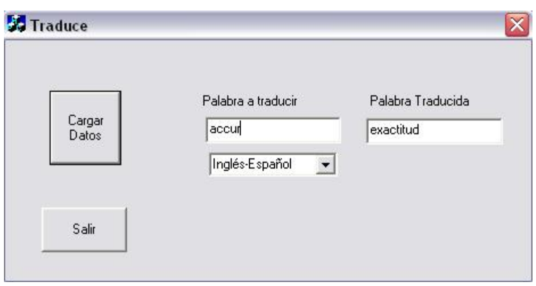


Fig. 8. The translator recalls correctly the word associated to "accuracy" even though it is not complete.

Now, asume that instead of writing a “y”, and “i” is keyed by mistake. The result is shown in figure 9.

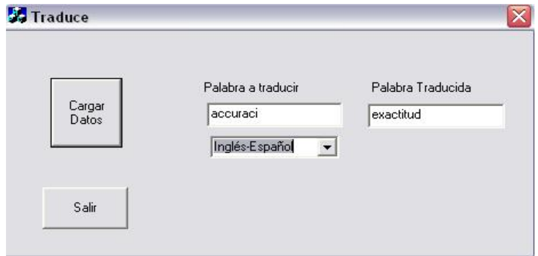


Fig. 9. Even where there is a writing mistake and a “y” is exchanged by an “i”, the program recalls in a correct manner the word “exactitud”.

We can see that in this example, a writing mistake, which at pattern level would be interpreted as introducing a noisy pattern, does not impede the performance of the translator.

The advantages presented by the translator reflect the advantages of the Alpha-Beta BAM model. These memories are immune to certain amount and kinds of noise, properties which still have not been characterized.

Besides, these kinds of tests, the full set of 120 words in English was entered, perfectly recalling their corresponding Spanish translations. Also, the 120 words in Spanish were written and, in a correct manner and without ambiguities, the translator showed the corresponding words in English.

4 Conclusions

Presented results show that the Alpha-Beta BAM model, introduced in this paper, has correct recall of all patterns in the fundamental set.

This correct recall requires no condition. The trained patterns do not need to fulfill certain properties for the Alpha-Beta BAM to be able to recall them in a perfect manner.

The algorithm of this BAM is not an iterative process and does not require convergence for its solution; also, it does not present any stability problem.

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