An Algorithm for Computing Design Parameters of IFIR Filters with Low Complexity

Algoritmo para el Cálculo de Parámetros de Diseño de Filtros IFIR de Baja Complejidad

Javier Díaz-Carmona[1], Gordana Jovanovic-Dolecek[2]and José A. Padilla-Medina[1].

[1] Technological Institute of Celaya, Ave. Tecnologico y G. Cubas, Postal Code 38010,
Celaya, Guanajuato, México.
email: jdiaz@itc.mx/apadilla@itc.mx.

[2] National Institute of Astrophysics Optics and Electronics, P. O. Box 51 and 216,
Postal Code 72000,
Puebla, Puebla, México.
email: gordana@inaoep.mx.

Article received on August 17, 2006; accepted on November 11, 2006

Abstract

This paper describes an algorithm for computing design parameters of Interpolated FIR (IFIR) digital filters with a low complexity (small number of products per output sample). The interpolation function is performed by a sharpened cascaded Recursive-Running Sum (RRS) filter, where the sharpening technique is used to improve the RRS frequency domain characteristics. Given the desired filter specifications and a chosen sharpening polynomial, the proposed algorithm computes the maximum IFIR filter interpolation factor and the minimum number of stages of the RRS filter in such a way that the overall structure meets the desired specifications with the minimum complexity.
Keywords: Narrowband filtering, Digital filters, IFIR, Sharpening technique, RRS filter.

Resumen

En este artículo se describe un algoritmo para calcular los parámetros de diseño de filtros digitales FIR interpolados (IFIR) con una complejidad baja (número pequeño de productos por muestra de salida). La función de interpolación se lleva a cabo mediante un filtro de suma en línea recursivo (RRS) moldeado, en donde la técnica de moldeo se aplica para mejorar las características en el dominio de la frecuencia del filtro RRS. Dadas las especificaciones deseadas del filtro y un polinomio de moldeo, el algoritmo propuesto calcula el máximo factor de interpolación del filtro IFIR y además el número mínimo de etapas en cascada del filtro RRS de manera que la estructura completa satisfaga las especificaciones con la mínima complejidad.
Palabras Clave: Filtrado de banda angosta, Filtros digitales, IFIR, Técnica de moldeo, Filtro RRS.

1 Introduction

There are many applications in digital signal processing systems where a digital filter with a narrowband and sharp transition band is required. The use of a Finite Impulse Response (FIR) digital filter in such applications exhibits a number of desirable properties such as guaranteed stability and linear-phase. However due to the large length required in practical applications, the FIR filters are difficult and sometimes impossible to be implemented using conventional structures

The interpolated FIR (IFIR) digital filter proposed by Neuvo et al. (1984) is one of the most promising approaches, [Mitra et al., 1993], for the design of low complex narrowband FIR filters and a rather well-developed topic in the field of digital filters [Mehrnia and Willson, 2004]. The IFIR filter $H(z)$ is a cascade of two filters

$$H(z) = G\left(z^{M}\right)I(z) \tag{1}$$

where $G(z^M)$ is an expanded model filter, $I(z)$ is an interpolation filter and M is the interpolation factor. The narrow band FIR prototype filter $H(z)$ is designed using lower length filters, $G(z)$ and $I(z)$.

A linear increase in the interpolation factor *M* results in the exponential growth of the interpolation filter order, as well as in the decrease of the model filter order. A key design point is to choose the right interpolation value *M* in order to meet the desired specifications with the minimum complexity.

In order to decrease the model filter order as much as possible a recursive-running sum (RRS) filter, [Mitra, 2005], can be used to perform the interpolation function [Jovanovic-Gordana and Diaz-Carmona, 2002]. The transfer function of an RRS filter with length *M* is given by

$$H_{RRS}(z)=\left(\frac{1}{M}\sum_{k=0}^{M-1}z^{-k}\right)^{L}=\left(\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right)^{L} \tag{2}$$

where *L* is the number of stages. The RRS filter is a linear-phase low pass filter that requires no multipliers and two additions per output sample, but it has a high pass-band droop and a low stop-band attenuation. The filter sharpening technique, [Kaiser and Hamming, 1984; Hartnett and Boudreaux, 1995; Samadi, 2000], can be used to improve the frequency characteristic of the RRS filter.

In this paper we propose an algorithm to find the interpolation factor *M* and the number of the stages *L*, for a given sharpening polynomial, so that the specifications are satisfied with as low as possible complexity. Next section describes in a briefly way the filter sharpening technique. The proposed algorithm is described in section three, followed with a design example.

2 Filter Sharpening

The sharpening technique, [Kaiser and Hamming, 1984; Hartnett and Boudreaux, 1995], attempts to improve the pass-band and/or stop-band of a linear-phase FIR filter by using multiple copies of the same filter. This technique is based on the use of a polynomial approximation which maps a transfer function $H_{RRS}(\omega)$, before sharpening, to a transfer function $H_{SpRRS}(\omega)$. The sharpening polynomial is obtained using the closed formula [Samadi, 2000]:

$$H_{SpRRS}(\omega)=\delta H_{RRS}(\omega)+\sum_{j=n+1}^{R}\left(b_{j,0}-\sigma b_{j,1}-\delta b_{j,2}\right)H_{RRS}^{j}(\omega), \tag{3}$$

where:

$$b_{j,0}=\sum_{i=n+1}^{j}(-1)^{j-i}\binom{R}{j}\binom{j}{i},\quad b_{j,1}=\sum_{i=n+1}^{j}(-1)^{j-i}\binom{R}{j}\binom{j}{i}\left(1-\frac{i}{R}\right),$$

$$b_{j,2}=\sum_{i=n+1}^{j}(-1)^{j-i}\binom{R}{j}\binom{j}{i}\frac{i}{R}.\quad R=n+m+1,\quad j=n+1,....R.$$

The design parameters are defined as:

- Parameter *n* is the tangency order to the line of slope δ at $[H_{RRS}(\omega),H_{SpRRS}(\omega)]=(0,0)$.
- Parameter *m* is the tangency order to the line of slope σ at $[H_{RRS}(\omega),H_{SpRRS}(\omega)]=(1,1)$.

In this way the polynomial coefficients are always integers, which can be implemented as Shift-and-Add multipliers, [Parhi, 1999]. For instance if both line slopes are set to zero and both tangency orders are one, then the sharpening polynomial is $H_{SpRRS}(\omega)=3H_{RRS}^{2}(\omega)$ $-2H_{RRS}^{3}(\omega)$, which graph, shown in Figure 1, denotes an RRS pass-band improvement. The corresponding structure for this example is shown in Figure 2. In general to avoid a fractional delay branch for any sharpening polynomial the factor $(M-1)L/2$ must be integer, which implies to avoid even interpolation factors for odd number of stages.

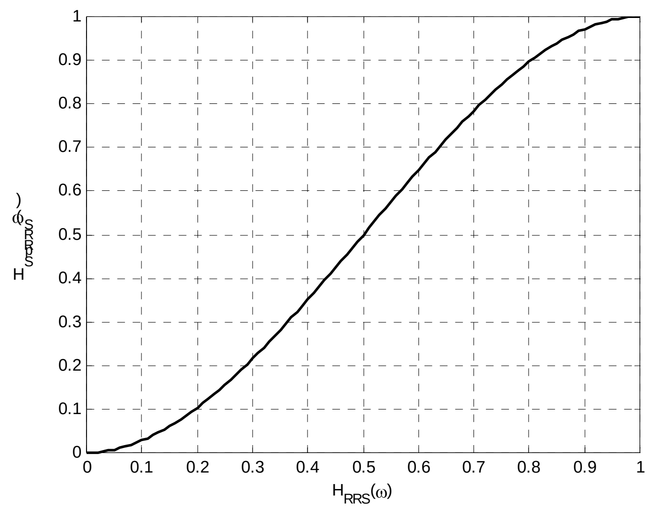

Fig. 1. Sharpening polynomial graph for $\sigma = \delta = 0$ and n = m = 1.

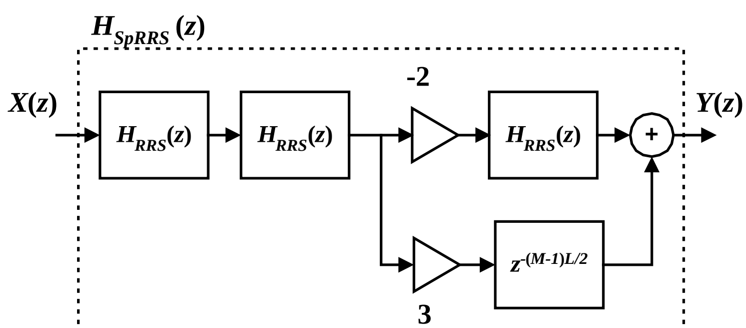

Fig. 2. Sharpening polynomial structure for $\sigma = \delta = 0$ and n = m = 1.

3 Proposed Algorithm

The algorithm is based on an iterative design of the model filter $G(z)$ using following specifications:

$$\begin{aligned} &\omega_{Gp} = M\omega_p, \ \ \omega_{Gs} = M\omega_s, \\ &\delta_{Gp} = \alpha\delta_p, \ \ \delta_{Gs} = \delta_s, \end{aligned} \tag{4}$$

where ω_{Gp} and ω_{Gs} are the normalized pass-band and stop-band frequencies, respectively, δ_{Gp} the maximum pass-band ripple and δ_{Gs} the minimum stop-band attenuation of the model filter $G(z)$, and ω_p, ω_s, δ_p and δ_s are the corresponding design specifications of the desired filter. The parameter α is a key factor in this design. The higher value of α the smaller model filter order, and vice versa. For a given value of M the maximum value of α is determined by the sharpened RRS filter pass-band droop. Our experience says that it is useful to take the values of α in the range $0.5 \le \alpha \le 0.9$.

In Figure 3 is shown the flow diagram of the proposed algorithm. At first step, the maximum possible interpolation factor, $M = M_{max} = \pi/\omega_s$ and the minimum possible number of RRS stages $L = 1$, are selected. The choice of M is a key factor to satisfy the pass-band specification, while the choice of L is a key factor to satisfy the stop-band specification of the designed overall filter. Consequently, in the next steps the factor M is increased until the pass-band specification is satisfied, interpolation factor searching loop, while the parameter L is increased until the stop-band specification is reached, RRS stages searching loop. If as a result, an even value of M and an odd value of L are obtained from these searching loops, the factor M is reduced by one to avoid a branch fractional delay in the sharpened RRS filter, [Webb and Munson, 1996]. In the following steps the algorithm looks for a possible further model filter order reduction by increasing the model pass-band ripple δ_{Gp}, while keeping satisfied the given overall filter specification.

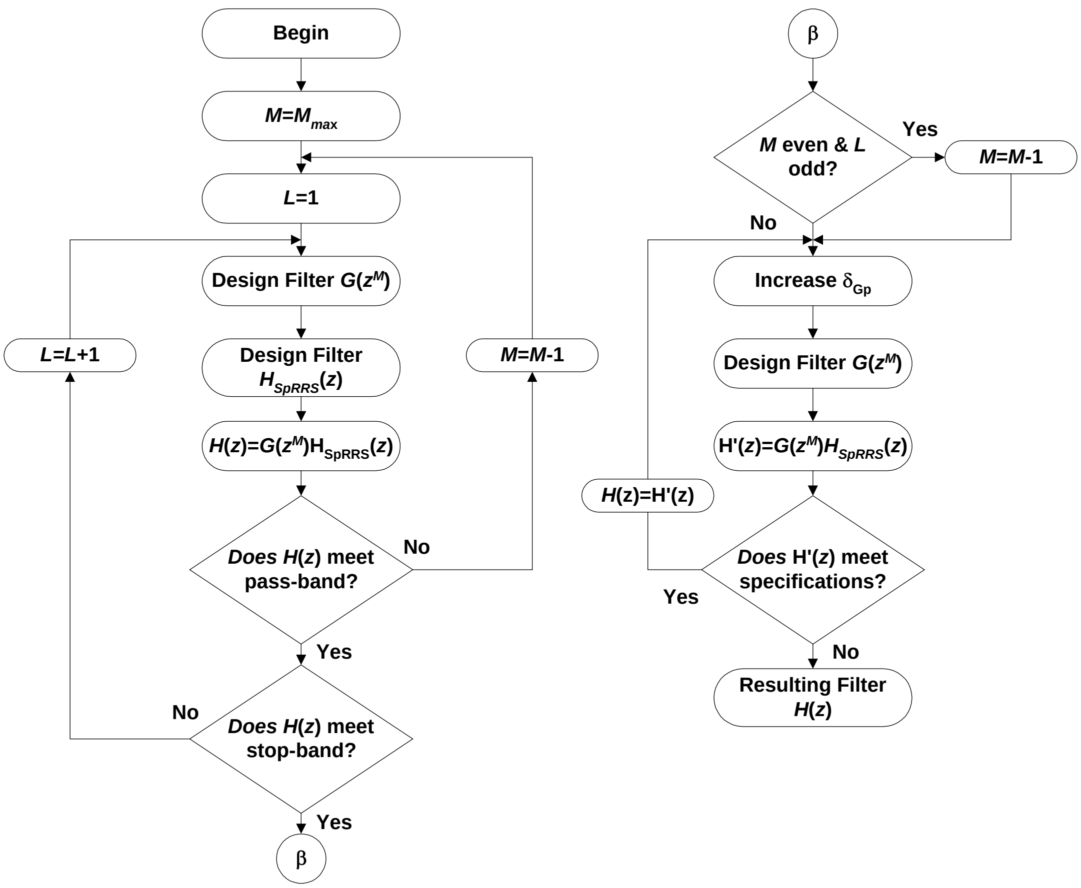

Fig. 3. Flow diagram of the proposed algorithm.

The proposed algorithm complexity, number of operations performed, is strongly determined by the interpolation factor and RRS stages searching loops, this is the number of iterations required to meet the desired pass-band and stop-band specifications, respectively. The main iteration computational work load is the design of both the model filter, by

ISSN 1405-5546

Parks McClellan algorithm, and the sharpened RRS filter, by the sharpening polynomial. Hence the total algorithm complexity depends on how fast the parameters are found for both pass-band and stop-band specifications. This speed is determined basically by two factors: 1.- Desired filter specifications: in this sense the narrower pass-band the smaller number of iterations in the interpolation factor searching loop, and the smaller stop-band attenuation the smaller number of iterations in the RRS stages searching loop. 2.- Sharpening polynomial structure: due the sharpening polynomial modifies the RRS pass-band and stop-band frequency magnitude response. For instance the higher the sharpening polynomial improves the RRS pass-band the smaller number of iterations in the interpolation factor searching loop, and the higher the sharpening polynomial improves the RRS stop-band the bigger number of iterations in the RRS stages searching loop.

Accordingly to the algorithm there may be next possible no convergence cases: 1.- Any interpolation factor M meets the pass-band specification. 2.- The increment of L in the RRS stages searching loop causes a not pass-band meeting for any M in the interpolation factor searching loop. If the design is limited to narrowband filters the first case is completely avoided and the second case is less prone to occur. Both no convergence cases are detected with a fixed maximum number of iterations.

4 Design Example

The described algorithm was implemented in MATLAB and different filters have been designed in a PC PIV at 1.7 GHz. We show as design example the low pass linear-phase FIR filter, proposed in Saramaki et al. (1988), with next specifications: ω_p=0.05π, ω_s=0.1π, δ_p=0.01, δ_s=0.001. According to the proposal by Mehrnia and Willson, (2004) the optimal interpolation factor M=4, results in the total number of products per output sample of 25 and the total number of additions of 47.

The design parameters obtained using the proposed algorithm for the sharpening polynomial with $\sigma=\delta=0$, $m=n=1$and $\alpha = 0.6$ are: $L= 2$, $M= 3$ with a number of floating point operations of 112885030 FLOPS and a computation time of 3.4 seconds. The model filter order is N_G = 35. The design parameters for the sharpening polynomial with $\sigma=\delta= 0$, $m= 2$, $n = 1$ and $\alpha= 0.6$ are: L=3, M= 5, a number of floating point operations of 106420351 FLOPS and a computation time of 4.0 seconds. In this case we have N_G= 21. In Figures 4 and 5 are shown the frequency magnitude responses for both cases.

As a matter of filter complexity comparison Table 1 shows the total number of products per output sample NPS_T, the total number of additions per out sample NAS_T, and the total delay units $NDel_T$ required for the proposed algorithm, and for algorithms proposed in [Saramaki et al.,1988], [Webb and Munson, 1996] and [Gustafsson et al., 2001]. Note that the smallest total number of products is obtained with the proposed algorithm using the sharpening polynomial with the parameters m= 2, n= 1. The method in [Saramaki et al., 1988] with 3 stages exhibits lower complexity than the proposed method with the sharpened polynomial m= n=1. However, the method [Saramaki et al., 1988] does not propose how to choose the design parameters.

Table 1. Filter complexity comparison between the proposed algorithm and other design methods.

Reference	Method	NPS_T	NAS_T	$NDel_T$
[Saramaki et al., 1988]	1 stage IFIR	18	34	119
	3 stages IFIR	15	24	127
[Webb and Munson, 1996]	Pseudo IFIR	24	46	122
[Gustafsson et al., 2001]	Two stages	20	36	90
	Three stages	24	45	120
Proposed Algorithm	m=n=1	18	50	125
	m=2, n=1	11	55	198

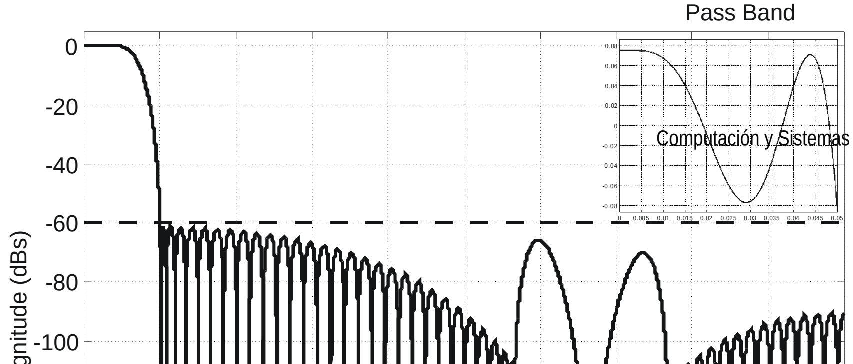

Fig. 4. Design example magnitude response obtained with the proposed algorithm with sharpening polynomial parameters $\sigma=\delta=0$, $m=n=1$.

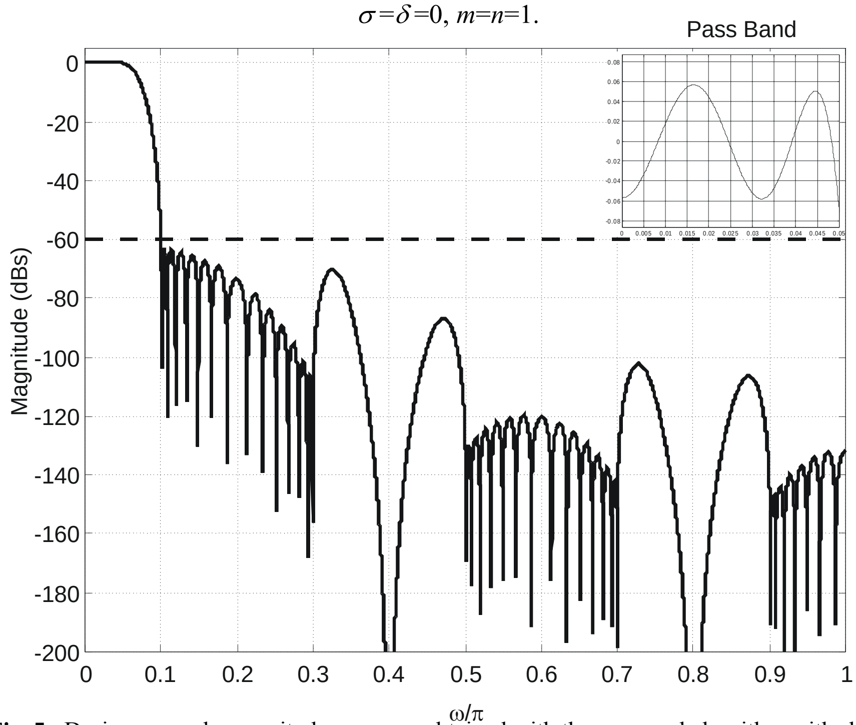

Fig. 5. Design example magnitude response obtained with the proposed algorithm with sharpening polynomial parameters $\sigma=\delta=0$, $m=2$, $n=1$.

ISSN 1405-5546

5 Conclusions

A direct algorithm to find the design parameters of an IFIR structure with a low complexity has been proposed, where the interpolation filter is a sharpened RRS filter. The proposal is based on an iterative design of the model filter $G(z)$ and the sharpened RRS filter until the filter desired specifications are met. As shown in the design example for a given sharpening polynomial, the proposed algorithm finds the maximum interpolator factor and the minimum number of RRS stages to meet desired specifications with small number of products per output sample. The proposed method is useful for narrowband FIR filter design.

References

1. **Gustafsson, O., H. Johansson** and **L. Wanhammar,** "Narrow-band and wide-band single filter frequency masking FIR filters." in IEEE International Symposium on Circuits and Systems, Sidney, Australia, 2001, pp. 181-184.
2. **Mehrnia A.** and **N. Willson Jr.,** "On optimal IFIR filter design." in IEEE International Symposum on Circuits and Systems, Columbia Canada, May, 2004, pp. 133-136.
3. **Mitra S.,** Digital Signal Processing: a Computer-based Approach, McGraw-Hill, New York, 2005.
4. **Mitra S., A. Mahalanobis** and **T. Saramaki,** "A generalized structural subband decomposition of FIR filters and its application in efficient FIR filter design and implementation." *IEEE Transactions on Circuits and Syst.-II: Analog and Digital Signal Processing,* Vol. 40, 1993, pp. 363-374.
5. **Neuvo Y., D. Cheng-yu** and **S. Mitra ,** "Interpolated finite impulse response filters." *IEEE Transaction on Acoustics Speech, and Signal Processing,* Vol. ASSP-32, 1984, pp. 563-570.
6. **Hartnett R. J.** and **G. F. Boudreaux ,** "Improved filter sharpening polynomial." *IEEE Transactions on Signal Processing*, Vol. 43, 1995, pp. 2805-2810.
7. **Jovanovic-Dolecek, G.** and **J. Diaz-Carmona,** "One structure for efficient narrowband bandpass FIR filters." in IEEE International. Midwest Symposium, Oklahoma, USA, August, 2002, pp. 485-488.
8. **Kaiser J. F.** and **R. W. Hamming,** "Sharpening the response of a symmetric nonrecursive filter by multiple use of the same filter." *IEEE Transactions on Acoustics Speech and Signal Processing*, Vol ASSP-32, 1984, pp. 415-422.
9. **Kwentus A. Y., Z. Jiang** and **N. Willson Jr. ,** "Application of a filter sharpening to cascaded integration-comb decimator filters." *IEEE Transactions on Signal Processing*, Vol. 45, 1997, pp. 457-467.
10. **Parhi K.,** VLSI Digital Signal Processing Systems Design and Implementation, John Wiley and Sons, New York, 1999.
11. **Samadi S.,** "Explicit formula for improved filter sharpening polynomial." *IEEE Transactions on Signal Processing*, Vol. 9, 2000, pp. 2857-2959.
12. **Saramaki T., Y. Neuvo** and **Mitra S.,** "Design of Computational Efficient Interpolated FIR Filters," *IEEE Trans. on Circuits and Syst*., Vol. 35, 1988, pp. 70-88.
13. **Webb J.** and **D. Munson,** "A new approach to designing computationally efficient interpolated FIR filters." *IEEE Transactions on Signal Processing*, Vol. 44, 1996, pp. 1923-1931.

Javier Díaz-Carmona *He received the B. S. degree in Electronics Engineering in 1990 from the Instituto Tecnologico de Celaya (ITC), Guanajuato, México, the Master in Science degree in 1997 and the Doctor in Science degree in Electronics in 2003 from the National Institute of Astrophysics Optics and Electronics (INAOE), Puebla, Mexico. Since 1991 he is part of the faculty of Electronics Engineering Department in the ITC, currently as full time professor. He is author of more than 20 papers. His research interests include Digital Signal Processing filter design techniques and Microcontoller, DSP/FPGA embedded systems applications. He is member of IEEE Signal Processing Society and National Researcher System (SNI) Mexico.*

Gordana Jovanovic-Dolecek *She received a BS degree from the Department of Electrical Engineering, University of Sarajevo, an MSc degree from University of Belgrade, a PhD degree from the Faculty of Electrical Engineering, University of Sarajevo, where she was professor until 1993, 1993-1995 she was at Institute Mihailo Pupin, Belgrade. In 1995 she joined INAOE, where she works as professor/researcher. During 2001-2002 and 2006 she was at Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher. She is editor of one book, author of three books and more than 200 papers. Her research interests include digital signal processing and digital communications. She is a Senior member of IEEE, Mexican Academy of Science, and National Researcher System (SNI) México.*

José A. Padilla-Medina *He received the B. S. degree in Electronics Engineering in 1991 from the Instituto Tecnologico de Celaya (ITC), Guanajuato, México, the Master in Science degree in 1995 from the University of Guanajuato, México and the Doctor in Science degree in Optics in 2002 from the Centro de Investigaciones en Óptica (CIO), Leon Guanajuato, México. Since 1995 he is part of the faculty of Electronics Engineering Department in the ITC, currently as full time professor/researcher. His research interests include Receiver Operating Characteristic (ROC) theory, Fuzzy logic and Fuzzy clustering applications and digital image/signal processing. He is member of National Researcher System (SNI) Mexico.*

ISSN 1405-5546