Qubits Structure as an Enhancement Factor of Coherence in a One-Way Quantum Computer

Estructura de Qubits como un Factor de Realce de Coherencia en una Computadora Cuántica de un Solo Camino

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Abstract

Present day's efforts for building up an operative quantum computer soon will take shape. One of the main challenges to this task is to implement qubit coherence in a practical way. We make emphasis on the structure of the nuclear qubits in a one-way quantum computer as a source of coherence enhancement. The form factor, accounting for the nuclear qubit structure of the model, is the magnetogyric ratio γ (more commonly called the gyromagnetic ratio). We collect experimental values for γ and calculate the respective times of coherence T^h, for a number of materials. A parametrization is also given for γ, in terms of the atomic number, whose agreement with the experiment is very good. We also calculate, accurately enough, bounds to the corrections to T^h due to spurious dipolar coupling between nuclei because this has not been done in the past. Such corrections are negligible for nearby planes whereas for remote planes they might be of considerable size. It is concluded that the nuclei states last longer than their electronic counterpart. However, this stability of nuclei qubits limits the speed at which the computer can carry out instructions and process the information.
Keywords: Qubit, structure, form factors, dipolar coupling, spurious, decoherence time.

Resumen

Esfuerzos presentes para construir una computadora cuántica operativa, pronto tomarán forma. Uno de los principales retos de esta tarea es el de implementar coherencia de qubits de una manera práctica. Hacemos énfasis en la estructura de los qubits nucleares en una computadora cuántica de un solo camino como fuente de realce de coherencia. El factor de forma, que explica la estructura del qubit nuclear, del modelo es la razón magnetogírica γ (más comúnmente llamada la razón giromágnetica). Colectamos valores experimentales para γ y calculamos los respectivos tiempos de coherencia, T^h, para diferentes materiales. Es también dada una parametrización para γ en términos del número atómico cuyo acuerdo con el experimento es bastante bueno. También calculamos cotas suficientemente exactas para las correcciones a T^h debidas a acoplamientos espurios dipolares en tanto que en el pasado no se les ha tratado con suficiente atención. Tales correcciones son despreciables para planos cercanos mientras que para planos remotos, ellas podrían ser de tamaño considerable. Es concluido que los estados nucleares duran más que su contraparte electrónica. No obstante, esta estabilidad de los qubits nucleares limita la rapidez a la cual la computadora puede efectuar las instrucciones y procesar la información.
Descriptores: Qubit, estructura, factores de forma, acoplamiento dipolar, espurio, tiempo de deco-herencia.

1 Introduction

Researchers are trying different approaches to see if it's possible to make large, practical computers that use the states of subatomic particles to store and process information. Altough it remains largely theoretical, there are efforts to build a spin based solid state quantum computer. This device would use the spin of atoms or subatomic particles as a qubit in order to represent bits of information. The spin of the nucleus of an atom can act like a tiny magnet, and depending on the orientation of the magnetic field, it represents a 1 or a 0. One challenge to making quantum computers is to preserve the fragile states of qubits. Energy from the environment can disturb the qubits, causing

them to decohere. The more intense the interaction with the environment, the less the nuclear qubits keep coherent. The longer a qubit remains coherent, the more operations a quantum computer can carry out. As is well known, there are several possible sources for nuclear qubit decoherence [1], namely: (a) magnetic fluctutations in dysprosium; (b) thermal currents in dysprosium; (c) fluctuations of paramagnetic impurities in the silicon; (d) thermal motion of the bridge; and (e) uncontrolled dipolar couplings bewteen nuclei. In the present work we shall consider only the last one. On the other hand, in the literature there have been reported several ways of producing coherent qubits out of different states [1]-[4]. Among them are the following: (i) the atomic nuclei in the molecules that make up a liquid; (ii) nuclei of the isotope silicon29 making up a solid; (iii) phosphorous atoms placed at regular intervals in a silicon chip. It is believed that the use of solids with strings of silicon29 would have many advantages over using liquids [2]. However, silicon29 qubits processor technologies would be very slow, since their clock speed is less than 100 kHz. The reason for this is that magnetic fields cause atomic nuclei to resonate at relatively low frecuencies.The radio signals used to control the qubits must match their low resonant frecuencies, which limits the speed at which the computer can carry out instructions and compile. Although proposed silicon-based quantum-computer architectures have attracted attention because of their promise for scalability and their potential for synergetically using the avalaible resources associated with the existing Si technologies infrastructure, in the present work we want to explore other possibilites with different materials. For instance, electronic and nuclear spins of shallow donors (e.g. phosphorous) are ideal candidates for qubits because of their long spin coherence times due to their limited interactions with their enviroments [4]. On the other hand, in the literature very little has been said about the structure of nuclear qubits and its effects on decoherence times. From the uncertainty principle it is easily concluded that the decoherence times are inversely proportional to the strength of interaction of the qubits with their enviroment. At the same time this strength depends on the size of the form factors which account for the structure of the nuclear qubits. In Refs. [1],[5],[6] the gyromagnetic ratio γ , is considered as being part of the hamiltonian but not systematically studied as a possible form factor. In the present work the gyromagnetic ratio itself is considered as a form factor. As a consequence of this assumption, the form factor should be seen then as a source of coherence enhancement in nuclear dipolar coupling. In order to proceed further, we collect experimental values for γ for several different materials and calculate their respective decoherence times. As a result it is found that nuclear qubits become more stable than electron-silicon qubits do. However, the processing time of a Nuclear Magnetic Resonance (NMR) quantum computer is longer than that of one made of electron-silicon. The downside of using electrons is that their quantum states are short-lived compared to the states of nuclei. The way we proceed is as follows: in section II a short review is given of the one-way quantum computer model. In this same section the way in which the gyromagnetic ratio intervenes as a source of coherence enhancement is underlined and corrections to T^h coming from the spurious dipolar coupling between nuclei are made. In section III a short discussion of the results is given.

2 One-Way Quantum Computer Model and Form Factors

The one-way quantum computer is a scalable model of a quantum computer which was proposed by Raussendorf and Briegel in Refs.[7]-[8], and contains many interesting features. According to this model, the entire resource for the processing of the information is provided initially by an entangled (cluster) state $\Phi > c$. Cluster states can be created in any system where there operates a quantum Ising interaction (at very low temperatures) between two-state particles in a lattice configuration. Thus, the interaction will be described by the hamiltonian
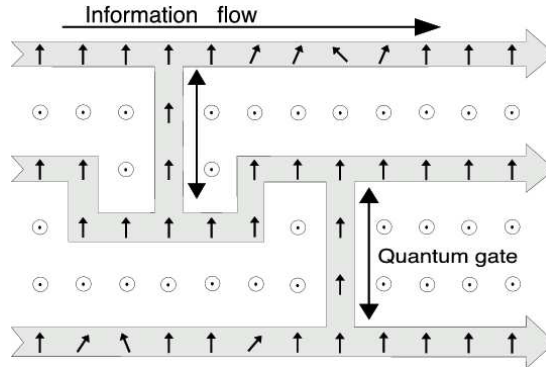
$$H = -\sum_{a,a'} J_{a,a'} \sigma_z^{(a)} \sigma_z^{a'}, \tag{1}$$

where $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is the z-component of the spin. In Ref. [8] it has been shown that the cluster state $\Phi > c$ is characterized by a set of eigenvalues equations

$$\sigma_z^{(a)}{}_{a'\in B(a)}\sigma_z^{(a')}\,|\varphi>c\pm|\varphi>c, \tag{2}$$

where $B(a)$ is a neighborhood with center at the qubit a.

Figure 1 depicts the way in which the quantum channels allow the propagation of information through a cluster.

Fig. 1. Quantum computation by measuring two-state particles on a lattice. Before the measurements the qubits are in the cluster state $\Phi>c$ of (2). Circles $\otimes$ symbolize measurements of σ_z, vertical arrows are measurements of σ_z, while tilted arrows refer to measurements in the $x-y$ plane

Assuming that the spin-spin coupling in (1) is of dipolar type, then it results [1]

$$J_{a,a'}=\hbar\delta\omega=\frac{\mu_o}{4\pi}\gamma^2\hbar^2\frac{1-3\cos^2\theta_{a,a'}}{r_{aa'}^3}, \tag{3}$$

where $r_{aa'}$ is the length [9] of the vector connecting the spins, $\theta_{aa'}$ is the relative angle between $r_{aa'}$ and the applied field, which satisfies $\cos\theta_{aa'}=2/3$, and γ is the magnetogyric ratio. Figure 2 sketches the way the regular chains of qubits are structured.

The magnetogyric ratios in Eq. (3) are given by

$$\gamma=\begin{cases}-\dfrac{g e \mu e}{\hbar} & electron\\[2mm]-\dfrac{g\mu_P}{\hbar} & nucleon,\end{cases} \tag{4}$$
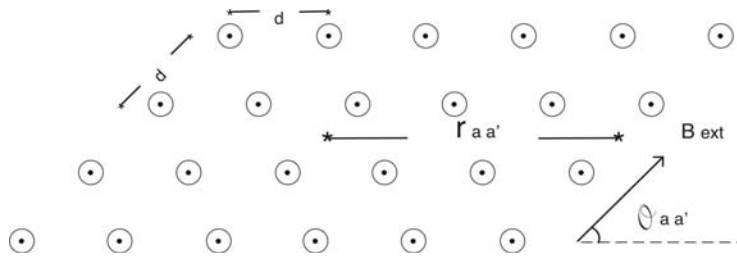
Fig. 2. Regular qubit chains define planes.

where $g_e \cong 2$ is the electron's g-factor, μ_B is the Bohr magneton, μ_P is the nuclear magneton, and g is the factor of the nucleon or nucleus in question. We may note that the above quantity contains the information concerning to the structure of the qubits. In other words, the magnetogyric ratio, γ, is the form factor of the model, and it is then one of the quantities to determine in the present work.

The electron magnetogyric ratio is given by NIST [10] as

$$\gamma_e = 1.76 \times 10^{11} \, Hz / Tesla. \tag{5}$$

Due to the fact that γ_e has a huge value, from Eq. (1) and (3) it follows that the electronic qubits have a very intense uncontrolled dipolar interaction. As a consequence, the scales of time where the electronic qubits states remain coherent, become negligible. Indeed, from Eqs. (3) and (5) it follows that

$$T^e = \frac{1}{\delta\omega} = 1.034 \times 10^{-5} \, sec. \tag{6}$$

Concerning the nuclear qubits, it is well known that there are several possible sources for their decoherence [1], namely: (a) magnetic flucutations in dysprosium; (b) thermal currents in dysprosium; (c) fluctuations of paramagnetic impurities in silicon; (d) thermal motion of the bridge; and (e) uncontrolled dipolar couplings between nuclei. In the present work, only the last one will be considered.

2.1 Decoherence by in-plane Dipolar Coupling
In this case, participating nuclei have equal Larmor frequencies and can thus participate in spin-flop processes. Therefore, acording to equation (3), decoherence is produced on a time scale of order [1]

$$T^h = \frac{4\pi\pi_{aa'}^{3}}{\gamma^2 \hbar \mu_o} = \frac{3.2021 \times 10^{17}}{\gamma^2}. \tag{7}$$

With the purpose of determining specific values for the times of decoherence of several different interesting nuclei, in the present work we collect experimental values for their gyromagnetic ratios. Once the values are fixed for γ, it is important to calculate their respective time scales of decoherence. In Table 1 are listed the nuclei, the experimental values of γ [11], and the respective decoherence times as they are given by Eq. (7)

Out of curiosity, it is worth it to observe from Table 1 that the following parameterization for γ is satisfied
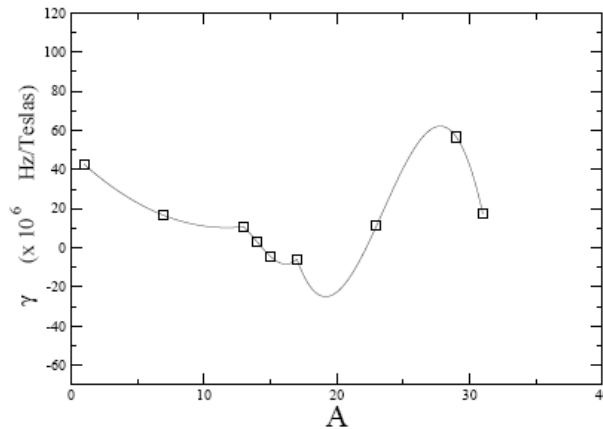
Fig. 3. Values of the form factor γ as predicted by the parameterization of Eq. (8). Squares represent the respective xperimental values taken from Ref. [11]

$$\gamma = \begin{cases} (2/7)(A-12)^2 + 152 & 1 \le A \le 13 \\ & 13 \le A \le 17 \\ (1/2)(A-14)^3 + 97(A-39) & 17 \le A \le 31 \\ -(3/10)(A-190/9)^3 - 33A - 28942 & \end{cases} \qquad (8)$$

where A is the atomic number. The parameterization of Eq. (8) is plotted in Figure 3 together with the experimental values. As can be appreciated from this figure, the fitting (8) is very good.

2.2 Decoherence by spurious Dipolar Coupling between nuclei in one homogeneous plane and copies of the nuclei in the next

This kind of coupling occurs when two qubits are recoupled for a logic gate, and they cause a small error in each logic gate. To estimate this error one qubit is recoupled into another qubit which is located n planes away ($n = 1,2,3,...$). The couplings from all of the neighbors may be treated as a T_2 decoherence process [1], with

$$T_2^3(n) = 4T^h n^3 \sqrt{\frac{1}{\sum_{a' \in B(a)} \left(\frac{\lambda_{a'}^2 / n^2 - 2}{\lambda_{a'}^2 / n^2 + 1} \right)}}$$

$$T_2^r(n) = T^h F(n)$$

$$n = 1,2,3,.....$$

$$(9)$$

where λ_a is the lateral distance to the *a*-th chain normalized to the distance, d, between chains. In this work, we are assuming a regular lattice where the distance between consecutive neighbors in a particular chain, is equal to d, as it is illustrated in Figure 2. Furthermore, we also assume here that $B(a)$ is a single chain containing the qubit a.

Table 1. Experimental values for γ taken from Ref. [11] and the respective T^h (as it is given by Eq. (7)) for several nuclei

Nucleus	$\gamma(\times 10^6\,Hz/Tesla)$	T^h sec
^{29}Si	56.587	100
^{1}H	42.576	154
^{7}Li	16.546	1169
^{13}C	10.7705	2760
^{14}N	3.0766	33829
^{15}N	-4.3156	17132
$^{17}?$	-5.7716	9612
^{23}Na	11.262	2524
^{31}P	17.235	1078

As a consequence of this last, $\lambda_a = 1,2,3,...,N$, being N the total number of entangled qubits neighbors to a, that is, the total number of qubits in the cluster satisfying Eq. (2). The function $F(n)$ represents the approximate error in the gate when attempting to couple qubits n planes apart. In Ref. [1], it is argued that in order to keep gate errors low, the computer should couple only nearby neighbors and handle more distant couplings by bit swapping. By this reasoning, gate errors due to unrefocused nuclear couplings between chains is limited to approximately $F(1) \cong 10^{-6}$. However, we consider that corrections to the coherence time by spurious dipolar coupling between nuclei, as given by Eq. (9), deserve major attention. Indeed, under the assumption that $B(a)$ is a single chain containing the qubit a one has

$$\frac{1}{2}\sum_{a'\in B(a)}\left(\frac{\lambda_{a'}^2/n^2-2}{\lambda_{a'}^2/n^2+1}\right)=\frac{1/n^2-2}{1/n^2+1}+\frac{4/n^2-2}{4/n^2+1}+\frac{9/n^2-2}{9/n^2+1}+...+\frac{N^2/n^2-2}{N^2/n^2+1}. \tag{10}$$

From the above expression, one obtains

$$\frac{2N}{N^2/n^2+1}\left[\frac{N(N+1)(2N+1)}{6n^2}-2N\right]\leq\sum_{a'\in B(a)}\left(\frac{\lambda_{a'}^2/n^2-2}{\lambda_{a'}^2/n^2+1}\right)\leq\frac{2N}{1/n^2+1}\left[\frac{N(N+1)(2N+1)}{6n^2}-2N\right]$$

$$\frac{2n^2N^2}{n^2+N^2}\left(\frac{N^2}{3n^2}-2\right)\leq\sum_{a'\in B(a)}\left(\frac{\lambda_{a'}^2/n^2-2}{\lambda_{a'}^2/n^2+1}\right)\leq\frac{2n^2N^2}{1+n^2}\left(\frac{N^2}{3n^2}-2\right), \tag{11}$$

where we have made the one-way quantum computer approximation [7],[13]: $N+1\approx N$. By comparing Eqs. (9) and (10), a constrain arises immediately, namely $\frac{N^2}{3n^2}-2\geq 0$. By using (11) in (9) we obtain the following bounds for

$T_2^r(n)$

$$2\sqrt{6n}\sqrt{1+n^2}\,\frac{\left(\dfrac{n}{N}\right)^2}{\sqrt{1-6\left(\dfrac{n}{N}\right)^2}} \leq \frac{T_2^r(n)}{T^h} \leq 2\sqrt{6n^2}\,\frac{\left(\dfrac{n}{N}\right)\sqrt{1+\left(\dfrac{n}{N}\right)^2}}{\sqrt{1-6\left(\dfrac{n}{N}\right)^2}} \tag{12}$$

As can be seen from the above expression, the maximal correction to the coherence time T^h by spurious dipolar coupling between remote planes (i.e. $n/N \to 1/\sqrt{6}$), can be important. However for relatively close planes ($n/N \ll 1$), the corrections are very small, i.e. $\approx 2\sqrt{6}n^3/N$.

3 Discussion

To contrast T^e, from Eq. (6), with the values for T^h given by Table 1, it can be concluded that the scales of times where the nuclear qubit states remain coherent are markedly larger than those of the electronic qubit states. The above property arises from the absence of structure (size) of the electrons. Tentatively, one might be tempted to conclude that nuclear qubit structure plays in favor of a quantum computer made of nuclei as an operative device. As we will see below, the last asseveration is not quite correct, although certainly, this is not completely false. In order to be more explicit, it must be observed that the nuclear qubits have the downside of having clock speeds of less than 100 kHz (i.e. they become very slow for the processing of the information). The reason for this is that the dipolar interaction causes atomic nuclei to resonate at relatively low frequencies. The radio signals used to control the qubits must match their low resonant frequencies, which limits the speed at which the computer can carry out instructions. In other words, when qubits are decoupled by a Hadamard scheme [1], the resulting pulse sequence has a clock time $t_{clock} = LN^2/\delta\omega = LN^2 T^h$ where $L = 400\,\mu m$ is a typical micromagnet length, N the number of qubits being decoupled and $L/\delta\omega$ is the amount of time devoted to one π pulse.[14] As qubits are added, some qubits in the chain become so distant that some $r_{aa'}$ grow and then from Eq. (3), $\delta\omega$ becomes very small. With this, t_{clock} becomes very large, that is, the respective quantum computer results very slow. For this reason, some research groups are working with electrons (with clock speeds of order 1 GHz [2]), rather than with atomic nuclei. Like nuclei, electrons act like tiny magnets, but they can be manipulated with ultrafast pulses of laser light. The disadvantage of using electronic qubits is that the electron quantum states are short-lived compared to the states of nuclei. An additional argument in favor of using nuclear qubits as quantum computers, is the infrastructure available for them at the moment. Notice that, from Table 1, of all of the nuclei considered here, ^{29}Si, has the smallest decoherence time (i.e. a larger clock speed). Therefore, ^{29}Si is one of the best candidates for a one-way quantum quantum computer. To conclude we note that the corrections to the coherence time by spurious dipolar coupling between nuclear qubits, found in the above section, can be important for remote planes (i.e. $n/N \to 1/\sqrt{6}$), and they cannot be trivially neglected. However, for relatively nearing planes ($n/N \ll 1$), the corrections are very small, since they are of order n/N.

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Glossary

Qubit: The logic register of a quantum processor. It can take the values $q = \alpha|0> + \beta|1>$ where $|0>$ and $|1>$ are the basic logic blocks and $\alpha^2 + \beta^2 = 1$.

Qubit structure: The qubit (nucleon) size.
Form factors: The mathematical functions accounting for the qubit size.
Dipolar coupling: Energy of interaction between the nuclei magnetic dipole.
Decoherence time: Time during which superposition of the states does not suffer quantum perturbation.

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