On Multicriteria Mixed Integer Linear Programming Based Tools for Location Problems-An Updated Critical Overview Illustrated with a Bicriteria DSS

Herramientas Multicriterio Lineales Enteras Mixtas para Problemas de Localización - Una Revisión Crítica Actualizada Ilustrada con un SAD Bicriterio

Maria Eugénia Captivo[1] and João Namorado Clímaco[2]

[1] Universidade de Lisboa, Faculdade de Ciências and Centro de Investigação Operacional, Campo Grande, Bloco C6, Piso 4, 1749-016 Lisboa, PORTUGAL
`mecaptivo@fc.ul.pt`

[2] Faculdade de Economia and INESC – Coimbra, Universidade de Coimbra, Av. Dias da Silva, 3004-512 Coimbra, PORTUGAL
`jclimaco@fe.uc.pt`

Abstract

Location problems are, in general, multidimensional in nature, particularly if sustainable development planning is required. So, multicriteria approaches seem adequate in many situations. Nevertheless, only a very small percentage of the publications in this area concern multicriteria models or tools. Generally, the different criteria are formulated as constraints imposing some minimum or maximum value, or are addressed by a surrogate criterion (like distance) on a single objective structure.

In this paper we outline the more relevant multicriteria mixed-integer location models and approaches taking into account several issues. The adequacy of the available models to reality is discussed. We also put in evidence the importance of interactive approaches, namely, discussing a decision support tool in which we are co-authors.

Keywords: Multicriteria Location, Mixed Integer Linear Programming, Interactive Approaches, Decision Support Systems

Resumen

Los problemas de localización son, en general, multidimensionales, sobre todo cuando se pretende desarrollar una planificación sostenible. De esta forma, los enfoques multicriterio son adecuados en muchas situaciones prácticas. Sin embargo, solo un pequeño porcentaje de publicaciones de esta área se focalizan en modelos o técnicas multicriterio.

En general, los diferentes criterios son introducidos en el modelo como restricciones que imponen algún valor máximo o mínimo, o que son enfocadas por un criterio sustituto (por ejemplo una distancia) en una estructura de apenas una función objetivo.

En este trabajo, hacemos una revisión de los modelos mixtos de localización multicriterio más importantes, así como de las técnicas que consideran algunas cuestiones importantes. También se evalúa la adecuación de los modelos existentes a la realidad. Destacamos la importancia de los planteamientos interactivos, en particular discutimos una herramienta de soporte a la decisión de la cual también somos autores.

Palabras Clave: Localización Multicriterio, Programación Lineal Entera Mixta, Aproximaciones Interactivas, Sistemas de Apoyo para la Toma de Decisiones

1 Introduction

In a Location Problem we want to locate a specific type of facility. Usually we look for the best way to serve a set of communities whose locations and demands are known. This implies to decide:

- the number and location of the facilities to serve the demand;
- the size or capacity of each equipment;
- the allocation of the demand points to the open facilities;

trying to optimize some objective function.

In general, location models are classified according to the optimality criterion and to the characteristics of the solution space.

Regarding the characteristics of the solution space, we can have a location problem on a network or on the Euclidean space. In each case we may consider a finite number of potential facility locations or an infinite one.

Perhaps continuous location models have been more studied but we will deal with discrete location models on networks due to its practical interest. Usually, potential locations for opening a facility are not available in any point of the Euclidean space. This is even truer if we are dealing with undesirable facilities.

Although we found several continuous location models for undesirable facilities in the literature, we believe that the location of this type of equipment should be analyzed as a discrete location problem on a network. The potential facility location sites are not available anywhere and we need to use the roads for the transportation between the communities and the open facilities.

Most location models deal with desirable facilities, such as warehouses, service and transportation centers, emergency services, etc., which interact with the customers and where usually travel is involved. As a consequence, typical criteria for such decisions include minimizing some function of the distances between facilities and/or clients (i.e., average travel time, average response time, cost function of travel or response time, maximum travel time/cost, etc.).

However, during the last two decades, those responsible for the overall development of the area, where the new equipment is going to be located (i.e., central government, local authorities) as well as those living there, are showing an increasing interest in preserving the area's quality of life.

Hence, new words have been introduced in the location theory, such as: noxious, obnoxious, semi obnoxious, hazardous, etc. As examples of undesirable facilities we can mention:
- nuclear and military installations;
- equipment emitting particulate or noise, warehouses containing flammable materials, regions containing refuse or waste materials;
- garbage dumps, sewage plants, correctional centers, mega-airports, …

The traditional optimality criterion of closeness (to locate the facility as close as possible to the customers) is replaced by the opposite criterion (how far away from the customers can the facility be placed ensuring accessibility to the demand points). This generates the NIMBY syndrome (NOT - IN - MY - BACK - YARD).

Frequently governmental regulations impose:
- minimum standards of quality;
- maximum levels of degradation;
- outright prohibitions.

Also, it cannot be forgotten that this type of problems involves group decision and negotiation among several decision actors.

The environmental issues on the approaches to undesirable facility location have generally been formulated as constraints or addressed by a surrogate criterion (distance) on a single objective structure. Nevertheless they deal with a number of conflicting objectives. As stated by Erkut and Neuman (1989), single objective models cannot be expected to accurately represent problems of this type. We agree with Current *et al*. (1990) when they say that the modeling of environmental issues as objectives, as opposed to constraints, would generate more information regarding the cost and other implications of environmental considerations.

Quite surprisingly the multiobjective decision making tools have barely been used in the undesirable facility location literature. Only a very small percentage of the publications in this area consider real multicriteria models or tools. The different criteria are formulated as constraints imposing some minimum or maximum value, or are addressed by a surrogate criterion (like distance) on a single objective structure.

In some papers we can find multiobjective location models [Malczewski and Ogryczak, 1990; Erkut and Neuman, 1992; Caruso *et al.*, 1993; Wyman and Kuby, 1993 and 1995; Melachrinoudis *et al.*, 1995], but in some cases the procedures used to solve them seem to be inadequate.

To deal with this type of models we can choose one of the following approaches:
- Calculation of the whole efficient set of solutions (generating methods);
- *A priori* articulation of preferences of the decision-maker (utility function methods);

- Progressive articulation of the decision-maker preferences (interactive methods) searching for a "satisfactory" efficient solution.

For this type of problem the number of efficient solutions can be very large. To present the decision-maker with all the solutions and to expect him/her to be able to choose a good one is not realistic.

In general we do not believe that the decision-maker has a process of defining an *a priori* utility function to be optimized.

We believe that interactive methods are the best choice, especially if they are thought as learning procedures (improving the knowledge about the problem) and not as procedures seeking some "optimal" solution. They should also be designed so as to be useful in a group decision and negotiation environment.

It should even be possible to do some *a posteriori* analysis around the preferred solution in order to select the one to be implemented, as presented in Figueira (1996). Making *a posteriori* analysis means analyzing in more detail, qualitatively indifferent solutions, in terms of the multicriteria models previously used. Eventually, some of these solutions can be "slightly" dominated in terms of the mathematical models used. As the model is not the reality but only an abstract representation of it, it makes sense to allow a more detailed study of the feasible region around the preferred solution.

Several multiobjective approaches for location of undesirable facilities consider that the set of potential locations was previously identified which, in fact, correspond to a common practice in real applications. These multiobjective discrete approaches imply two major difficulties: on one hand, the need of dealing with multidimensionality, and on the other hand, the large number of efficient solutions to find and analyze. As a consequence the use of interactive methods becomes of great importance.

We will discuss a bicriteria mixed integer linear model for facility location with environmental consequences and a decision support system based on an interactive procedure used to solve the model and analyze the solutions. Furthermore, uncertainty is also a key issue in location problems. As it is well known, multicriteria approaches are adequate, in these circumstances, to identify the criteria related to the stable part of the decision-maker preference structure [Bouyssou, 1989]. So, *a posteriori* detailed analysis of some satisfactory solutions (selected according to the decision-maker using a multicriteria mathematical programming approach) must be done in many cases.

Here we discuss several multicriteria location approaches taking into account several issues. In particular, a confrontation between some approaches available in the literature and one in which we are co-authors, is made. Finally, we will try to outline some future trends for research in this area.

2 Reviewing of Some Approaches to Multicriteria Facility Location

We are going to restrict our attention to the articles with a Multiple Objective Mathematical Programming formulation ("multiattribute" models, for example, are outside the scope of this paper). The multiattribute approaches enable a more detail study of a small number of explicitly known solutions. However they are unable to take into account all the possible solutions to a problem. In conclusion, more than alternative ways of analysis, in our opinion, they can be complementary.

Since 1978 we can find some approaches where more than one criterion is considered even if the algorithms proposed are only for the optimization of a weighted sum of the different objectives. In other cases, only one objective function is considered and the other criteria are incorporated in the model as constraints.

It was during the last decade that the majority of the papers in this area appeared. We could find some real applications regarding the location of hospitals, crèches, kindergartens, elementary schools, homes for the aged, sanitary landfills, hazardous materials facilities, etc. We will briefly mention some multiobjective location approaches already presented, particularly for equipments with environmental implications, but only interactive ones. In Malczewski and Ogryczak (1990) the location of hospitals (a real application in Warsaw) is formulated as a multiobjective optimization problem and an interactive approach DINAS [Ogryczak *et al.*, 1989] based on the so-called reference point approach [Wierzbicki, 1982] is presented. A real application is presented, considering 8 sites for potential location and at least 4 new hospitals to be built, originating 163 alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision-making process making the final decision less subjective. They also observed that during the

interactive process the decision-makers have gradually learned about the set of feasible alternatives and the consequences of possible decisions. As a consequence of this learning process they have changed their preferences and priorities.

Erkut and Neuman (1992) present a multiobjective mixed integer linear model for undesirable facility location. The objectives considered are total cost minimization, total opposition minimization and equity maximization.

Caruso *et al.* (1993) present a model for planning an Urban Solid Waste Management System considering the last three phases of a well known scheme structured into four phases: collection, transportation, processing and disposal. The technologies of incineration, composting and recycling are considered for the processing phase and sanitary landfills are considered for the disposal phase. Heuristic techniques (embedded in the reference point approximation) are used to solve the model and, as a consequence, "approximate Pareto solutions" are obtained (as named by the authors). By varying the reference point, different solutions can be obtained. The results for a case study (in Italy, for the Lombardy region) are presented and discussed.

Wyman and Kuby (1993, 1995) present a Multiobjective Mixed Integer Programming Model for the location of hazardous material facilities (including the technology choice variables) with 3 objective functions (cost, risk and equity).

Melachrinoudis *et al.* (1995) propose a dynamic (multiperiod) multiobjective capacitated mixed integer programming model for the location of sanitary landfills.

Fonseca and Captivo (1996, 2006, and 2007) study the location of semi obnoxious facilities as a discrete location problem on a network. Several bicriteria models are presented considering two conflicting objectives, the minimization of the obnoxious effect and the maximization of the accessibility of the communities to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance (or to its square) between demand points and open facilities and directly proportional to the population in each community. All the models are solved using Chalmet *et al.* (1986) non-interactive algorithm for Bicriteria Integer Linear Programming modified to an interactive procedure by Ferreira *et al.* (1994). Several equity measures are computed for each non-dominated solution presented to the decision-maker, in order to increase the information available to the decision-maker about the set of possible solutions.

Ferreira *et al.* (1996) present a bicriteria mixed integer linear model for central facilities where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira *et al.* (1994) is used to obtain and analyze non-dominated solutions.

Ferreira (1997) also presents a bicriteria mixed integer linear model for the location of semi obnoxious facilities incorporating the routing phase, considering as objectives the minimization of total cost and the minimization of the obnoxious effect of the open facility and the risk associated with the transport phase.

Giannikos (1998) presents a multiobjective discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centers that produce the waste and the candidate locations for the treatment facilities.

Cappanera *et al.* (2004) present a model for the problem of locating semi obnoxious facilities and simultaneously routing the undesirable materials between communities and facilities.

Dias *et al.* (2003a and 2006) propose the development of a DSS for dynamic location problems. Three types of facilities are considered: landfills, transfer stations and incinerators.Rakas *et al.* (2004) develop a bicriteria model for the location of undesirable facilities such as landfills. To reduce the number of landfill candidate sites these authors use a multiple attribute decision-making technique.

Haastrup *et al.* (1998) develop a DSS for waste management in a province of Sicily, allowing for the generation and evaluation of proper alternatives especially concerning environmental consequences.
Lahdelma *et al.* (2002) describe a real-life application of an ordinal multicriteria method to choose the location for a waste treatment facility in a region of Finland.

Costa *et al.* (2008) develop two bicriteria models for single allocation hub location problems. In both models total cost is the first criteria to be minimized. Instead of using capacity constraints to limit the amount of flow that can be received by the hubs, a second objective function is introduced in the models, trying to minimize the time to

process the flow entering the hubs. In the first model, total time is considered as the second criteria and, in the second model, the maximum service time for the hubs is minimized. Non-dominated solutions are generated using an interactive decision-aid approach developed for bi-criteria integer linear programming problems. Both bi-criteria models are tested on a set of instances, analyzing the corresponding non-dominated solutions set and studying the reasonableness of the hubs flow charge for these non-dominated solutions.

3   A Bicriteria Location Model

Let us consider the Bicriteria Simple Plant Location Problem (**BSPLP**), which can be formulated as:

$$\text{Min} \quad F_1(x, y) = \sum_{i \in I} \sum_{j \in J} l_{ij} x_{ij} + \sum_{i \in I} h_i y_i$$

$$\text{Min} \quad F_2(x, y) = \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} + \sum_{i \in I} g_i y_i$$

$$\text{s. t.:} \quad \sum_{i \in I} x_{ij} = 1 \qquad\qquad j \in J$$

$$y_i \geq x_{ij} \qquad\qquad i \in I, j \in J$$

$$y_i \in \{0,1\} \qquad\qquad i \in I$$

$$x_{ij} \in \{0,1\} \qquad\qquad i \in I, j \in J$$

where:

$J = \{1, \ldots, N\}$ is the set of clients or communities to be served,

$I = \{1, \ldots, M\}$ is the set of possible service locations,

h_i, g_i are fixed costs or values of opening service i,

l_{ij}, d_{ij} are transportation costs or values from assigning service i to client j,

and the variables can be defined as:

$$y_i = \begin{cases} 1 & \text{if service i is open} \\ 0 & \text{if service i is closed} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if client j is assigned to service i} \\ 0 & \text{if client j is not assigned to service i} \end{cases}$$

Besides the cost it also considers the minimization of the obnoxious or disagreeable effect. It seems to be suitable and simple enough to be accepted as relevant by the decision-maker and other actors, possibly associated with the decision process.

There are several examples in the literature [Ross and Soland, 1980; Hultz *et al.*, 1981; Revelle and Laporte, 1996] where different meanings for the objective functions shown can be found. Considering the location of undesirable facilities, one of the objectives usually represents total costs, and the other one total risk or noxiousness resulting from open services and transportation between clients and services. For instance, if h_{ij} represents the

noxious or detrimental effect on location j of a facility located at i, then $h_i = \sum_j h_{ij}$ can measure the total noxious effect of locating a facility at i. The noxious effect relative to the transportation of, for instance, undesirable materials between i and j can be represented by l_{ij}.

The two objective functions considered could be different from the ones shown as long as their weighted sum results in a linear objective function of the location variables y_i and the assignment variables x_{ij}. Additional constraints can also be considered (constraints on the number or capacity of the open facilities, budget constraints, …).

4 An Interactive Approach to Bicriteria Facility Location

A decision support system inspired in Ferreira *et al.* (1994) interactive procedure was developed to solve this type of bicriteria location models. The interactive method is based upon the progressive and selective learning of the non-dominated solutions set [see Clímaco *et al.*, 2003; Ferreira *et al.*, 1996]. As in any interactive method a computation phase, a dialogue phase and a stopping rule are considered (see Figure 1).
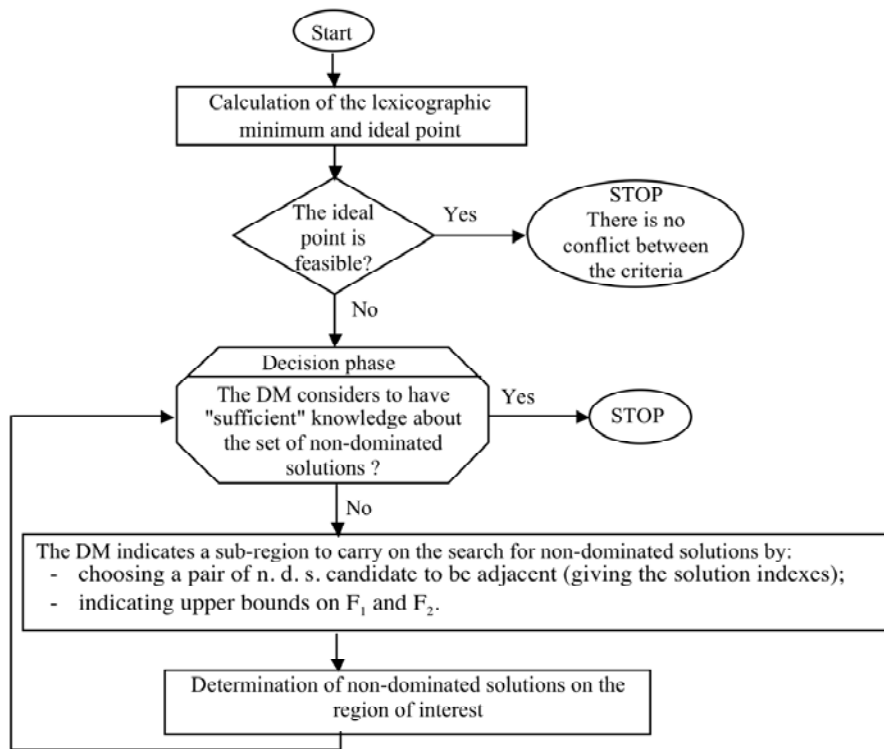

Fig. 1. General diagram of an interactive procedure

In the calculation phase we optimize a single criterion problem representing the weighted sum of both objective functions, imposing limits on their values accordingly to the preferences expressed by the decision-maker during the dialog phase. The weights are just operational parameters that can be fixed by the analyst or even by default using equal weights.

In discrete problems, the optimization of weighted sums of the different objectives only allows for the computation of the supported non-dominated solutions set. To obtain all non-dominated solutions (supported and unsupported) of a bicriteria integer linear model, extra constraints imposing limits on the values of the objective

functions must be included [Ross and Soland, 1980]. Obviously, some unsupported non-dominated solutions can be the most appropriate for the decision-maker(s). So, this sub-set of non-dominated solutions must be considered.

For the computation phase efficient algorithms should be chosen because time is quite important if we really want an interactive procedure. In the dialogue phase special care must be taken with the information required from the decision-maker. Also, the user-computer interface should be attractive and easy to use. In the bicriteria case much can be done in this topic.

The dialogue with the decision-makers regarding the elicitation of their preferences, concerns, by cognitive reasons, the objective function values, as pointed out by Larichev and Nikiforov (1987). This procedure seems to be easily understood by the decision-maker.

A general solver for mixed integer linear programming (for instance CPLEX or MATLAB) can be used to solve this single criterion problem. However, in many cases, there exist special algorithms dedicated to each problem taking advantage of its special structure. For the particular BSPLP model presented above, the special purpose algorithm of Dias *et al*. (2003b) was considered. Of course, in terms of computational efficiency, those are much better than general solvers. So, different algorithms to solve the single criterion problem can be incorporated in the system.

An interactive decision support tool incorporating all these procedures was implemented in Visual C^{++} [Fernandes *et al*., 2007]. It was developed in a modular way, allowing for the introduction of more models relevant in practical situations and/or procedures to solve them.

As stopping criteria, contrary to the use of a certain number of interactions, we prefer an open procedure where we only stop when the decision-maker considers having sufficient knowledge about the problem to be able to choose one of the solutions that were generated. No irrevocable decisions are made. It is always possible to go back and analyze some region that was previous discharged.

5 Example

We will illustrate the approach discussed above with a random generated instance with 10 communities and 5 potential facility locations. The first objective minimizes total risk or noxiousness resulting from the open facilities and the transportation between clients and facilities and the second objective minimizes total cost. The coefficients used in both objective functions are presented in Figure 2 and the network considered is shown in Figure 3.

$$h_i = \begin{bmatrix} 25 & 15 & 17 & 20 & 11 \end{bmatrix} \qquad g_i = \begin{bmatrix} 792 & 759 & 656 & 29 & 580 \end{bmatrix}$$

$$l_{ij} = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 2 & 1 \\ 4 & 2 & 2 & 3 & 1 \\ 2 & 1 & 2 & 2 & 1 \\ 3 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 \\ 4 & 2 & 2 & 3 & 1 \\ 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 & 1 \\ 3 & 2 & 2 & 3 & 1 \end{bmatrix} \qquad d_{ij} = \begin{bmatrix} 1086 & 232 & 641 & 1618 & 1553 \\ 1172 & 141 & 1014 & 1796 & 1462 \\ 1535 & 2133 & 2224 & 936 & 1610 \\ 1036 & 2117 & 1725 & 1660 & 3368 \\ 1374 & 1650 & 2063 & 750 & 1259 \\ 1737 & 706 & 1511 & 1694 & 2027 \\ 163 & 1150 & 526 & 787 & 2169 \\ 936 & 377 & 791 & 1560 & 1698 \\ 1589 & 1653 & 2278 & 965 & 2974 \\ 1814 & 2895 & 2503 & 2438 & 4146 \end{bmatrix}$$

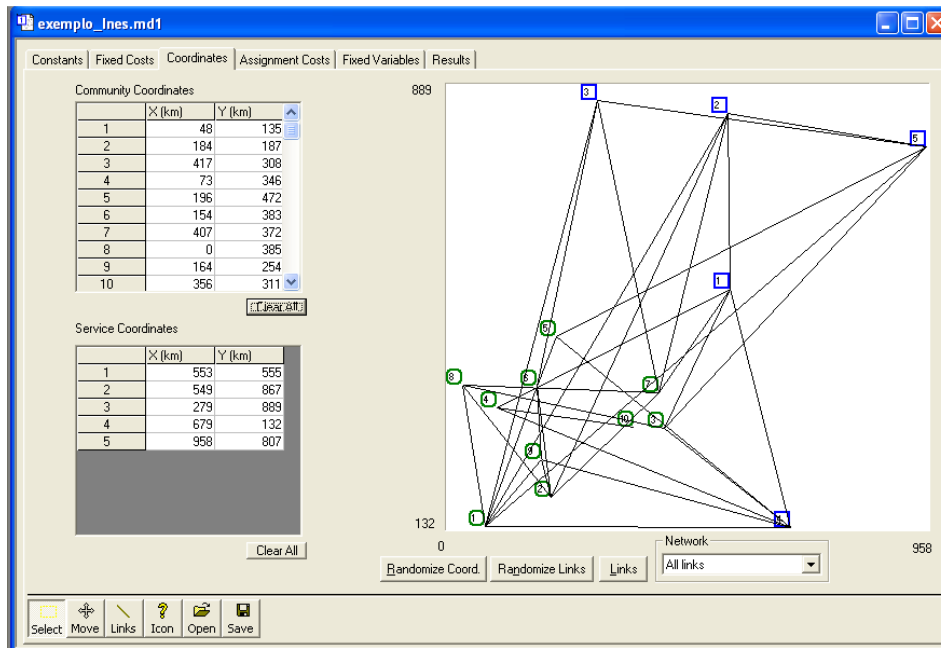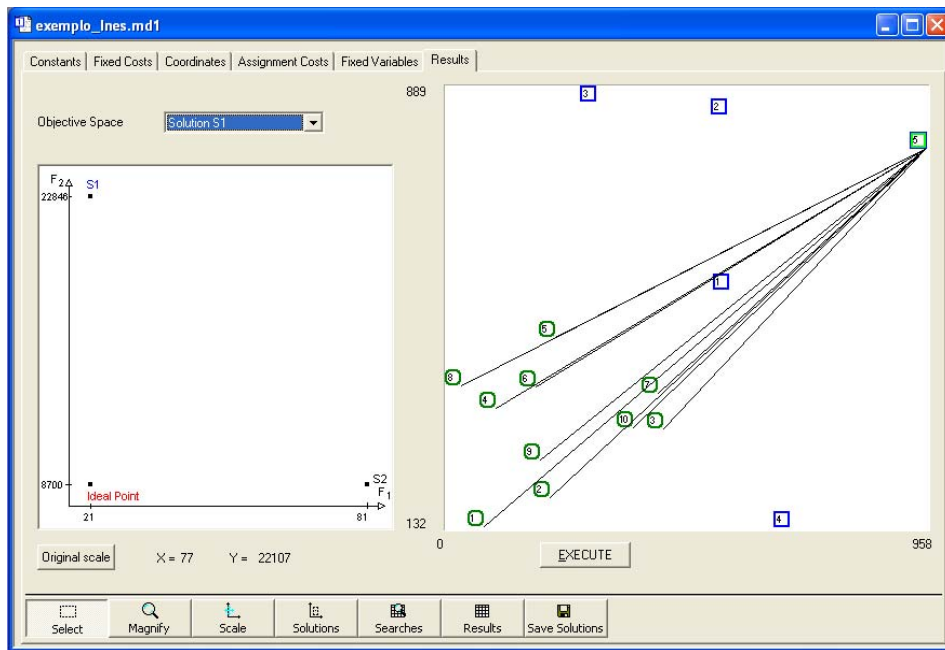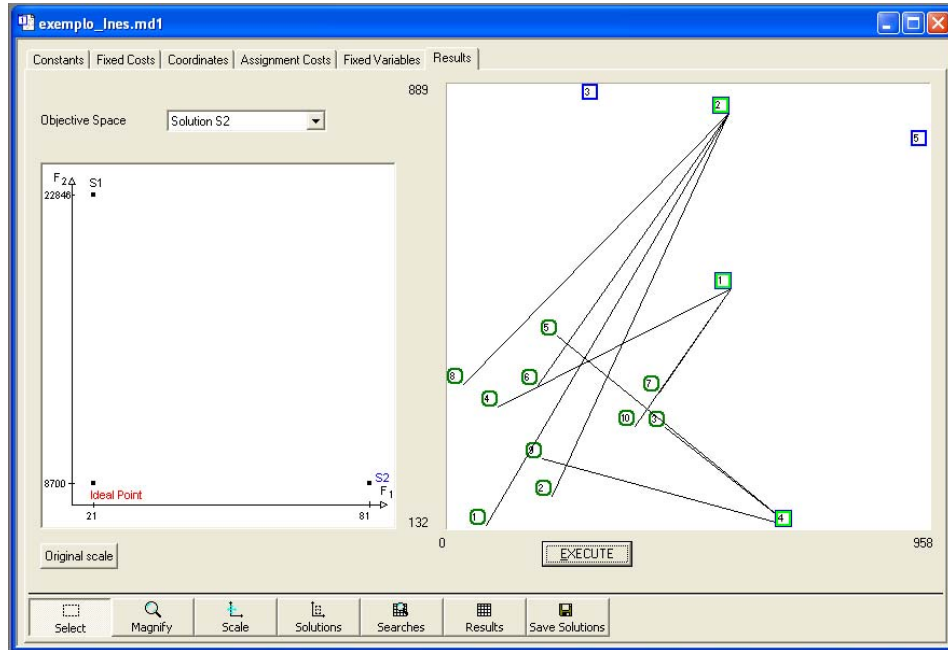**Fig. 2.** Coefficients used in both objective functions of the example

Fig. 3. Network considered in the example

The lexicographical optimum for each of the objective functions are $F^{(1)} = (21, 22846)$ and $F^{(2)} = (81, 8700)$. In the objective space (Figure 4) we can see the ideal solution and the lexicographical minimum for the objective functions.

a)

Fig. 4. Ideal solution and the lexicographical minimums in the objective space

Let us suppose that in the dialogue phase the decision-maker decides to indicate a sub-region where to search for non-dominated solutions by indicating upper bounds for the value of the objective functions $F_1 \leq 50$ and $F_2 \leq 15500$. This corresponds to select in the objective space the area marked in Figure 5.
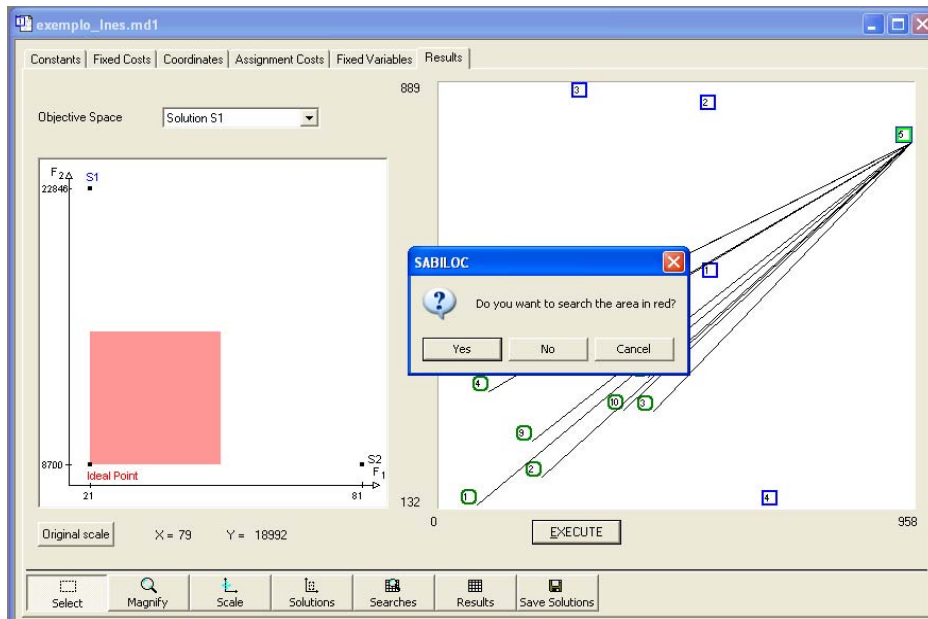

Fig. 5. Region to explore for non-dominated solutions

The problem to be solved is
$$\begin{array}{l} \min\ \lambda_1 F_1(x) + \lambda_2 F_2(x) \\ \text{s.t:}\quad F_1(x) \leq 50 \\ \qquad\ F_2(x) \leq 15500 \\ \qquad\ x \in X \end{array}$$
, where $x \in X$ is the original set of constraints and the

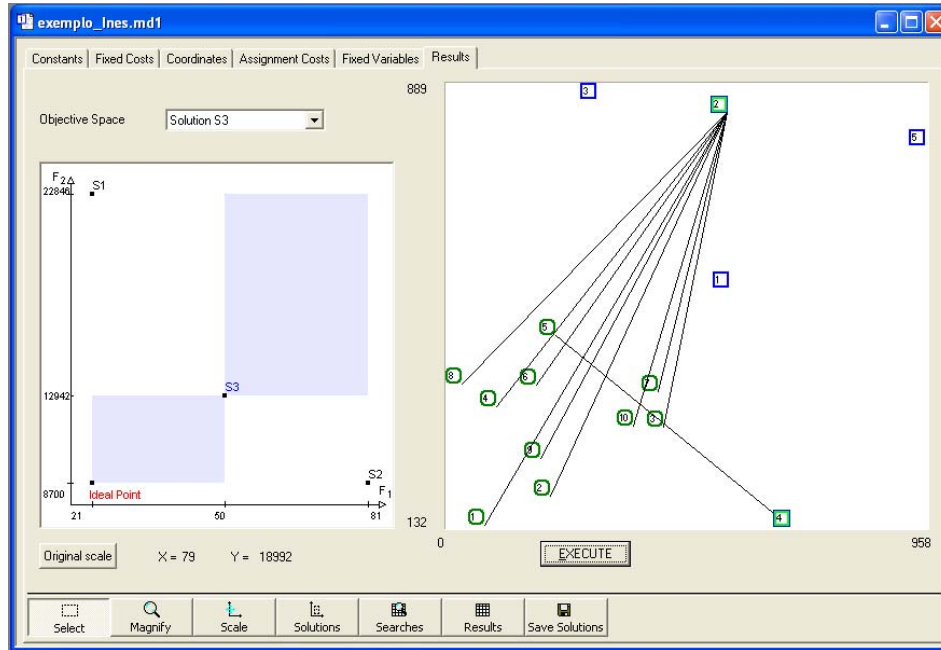optimal solution obtained (solution S3 presented in Figure 6) has $F^{(3)} = (50, 12942)$.

Fig. 6. Solution S3 and regions eliminated from the search in the objective space

We should note that the choice of the weights $(\lambda_1,\ \lambda_2)$ to use is not important. We can use any set of weights as long as the usual conditions $\lambda_1 + \lambda_2 = 1$ and $\lambda_1, \lambda_2 > 0$ are verified. Of course we can also choose the weights in order to try to obtain a non-dominated solution closer to $F^{(1)}$ or to $F^{(2)}$. Now, it is possible to reduce the region in the objective space where non-dominated solutions can still exist, as it is shown in Figure 6.

Let us suppose that, in the dialogue phase, the decision-maker decides to indicate a sub-region where to search for non-dominated solutions by choosing the pair (S3, S2) of potentially adjacent non-dominated solutions. This corresponds to select in the objective space the rectangular area between S3 and S2.

In this case the problem to be solved is
$$\begin{array}{l} \min\ \lambda_1 F_1(x) + \lambda_2 F_2(x) \\ \text{s.t:}\quad F_1(x) \leq 80 \\ \qquad\ F_2(x) \leq 12941 \\ \qquad\ x \in X \end{array}$$
, and the optimal solution obtained (S4) has

$F^{(4)} = (79, 9687)$, as it is shown in Figure 7. Again, we see a reduction in the region of the objective space where to look for non-dominated solutions.
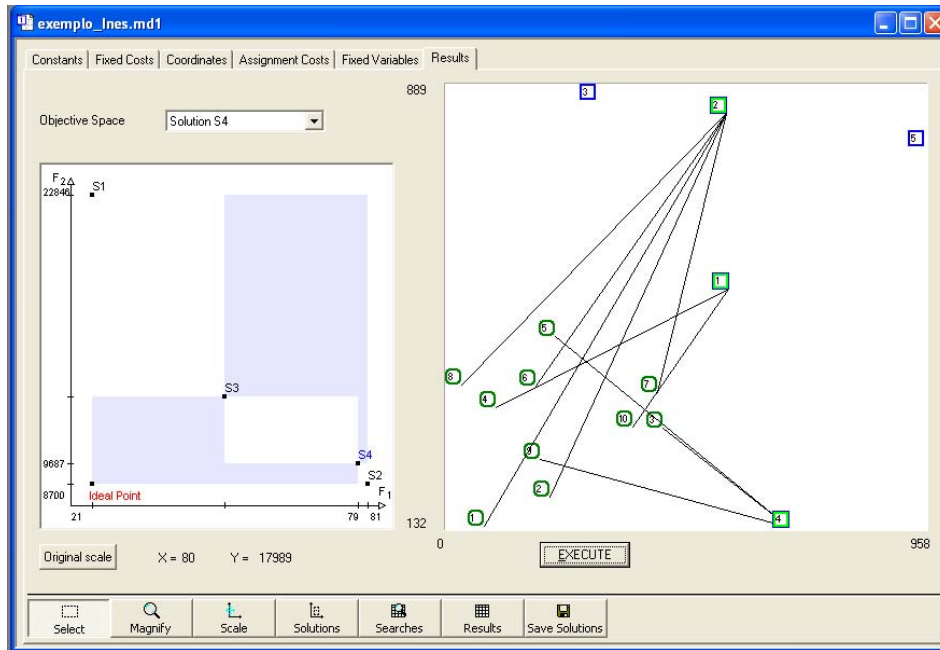
Fig. 7. Solution S4 and regions eliminated from the search in the objective space
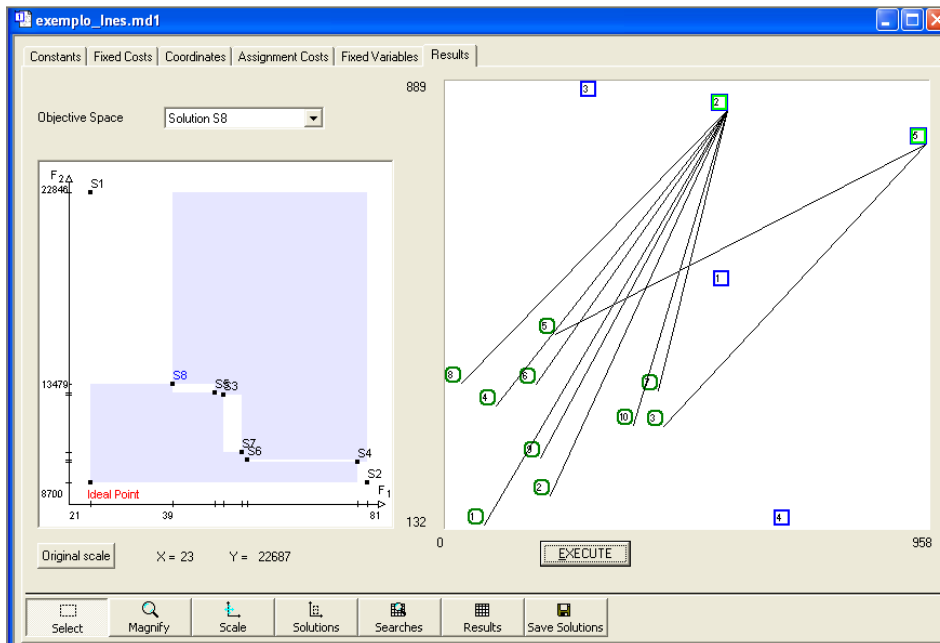

Fig. 8. Regions eliminated from the search in the objective space after 8 interactions

The information available in this graphical display is very useful to the decision-maker. Knowing the regions already explored the decision-maker searches for non-dominated solutions in a selective way, avoiding regions

where there are no non-dominated solutions, and improving his/her knowledge of the non-dominated solution set in a progressive way.

This type of graphical tools seems appropriate to look for compromises in situations where group decision should take place, eventually in the presence of hierarchies, facilitating the negotiation between different parts.

The procedure will continue in the same way, until the decision-maker considers having sufficient knowledge about the set of non-dominated solutions. After eight interactions the regions already explored in the objective space are shown in Figure 8.

If the decision-maker never considers having sufficient knowledge about the set of non-dominated solutions, the procedure will generate all the non-dominated solutions (14 in this example) ending with all the objective space already explored, as it is shown in Figure 9.
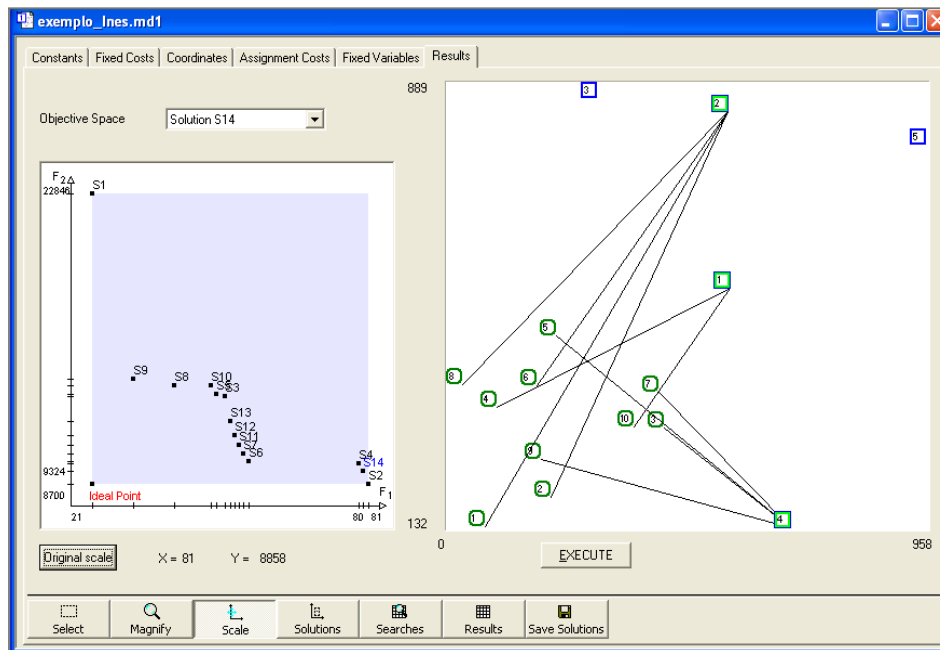

Fig. 9. Non-dominated solutions in the objective space

This figure also shows the importance of been able to generate all non-dominated solutions, and not only the supported ones. In this instance, from the 14 non-dominated solutions, only 4 are supported. All the other 10 are non-supported non-dominated solutions. We found this to be a common feature of bicriteria location problems. A great majority of the non-dominated solutions are non-supported.

6 Conclusions

It must be remarked that even for bicriteria problems (qualitatively simpler than the general multicriteria situations), both generating methods and interactive approaches are computationally much more demanding than the methods for single criteria optimization problems.

The bicriteria model is an important particular case in Multicriteria Optimization and, for that reason, has been considered in the literature, either from the point of view of practical applications or regarding theoretical studies [Clímaco *et al.*, 1997]. Of course it can be handled by general procedures for the *n*-dimensional case; nevertheless, the special structure of the bicriteria problem, particularly the advantage of the easy way to obtain a graphical

representation of the efficient frontier [Zeleny, 1974], soon gave place, in some way, to the separation of this class (with two objective functions) from the others (with more than two objective functions).

Some attempts to design interactive procedures for the same type of problems, but considering more than two criteria, are presented in Coutinho-Rodrigues *et al.* (1997). However there is no evidence that the tools used can accomplish so effective decision aiding.

The approach used here for to this type of BILP problems, presents some advantages when compared to other approaches:

i) Any non-dominated solution, including those in the duality gaps, can be determined.

ii) It is potentially useful for bicriteria models where great number of solutions may exist, as in the model presented above. It must be remembered that there are no irrevocable decisions along the process; the information required from the decision-maker in each interaction is simple; the process stops when the decision-maker considers that he/she got enough information about the non-dominated solution set.

iii) It is not bad from the operational point of view. In each interaction an integer linear programming problem is solved, and its original structure was altered, just because two additional constraints were included.

iv) In each interaction one obtains a new non-dominated solution (enabling the elimination of some sub-area of search from now onwards) or the whole area selected for search in the current interaction is eliminated.

This interactive approach has been used in several discrete linear programming models, namely in set-covering problems [Ferreira *et al.*, 1994, 1997], location-allocation problems [Ferreira *et al.*, 1997], and location-allocation-distribution problems [Ferreira, 1997].

General-purpose solvers were used for solving monocriteria instances in each interaction. However, in many cases, there exist special algorithms dedicated to each problem taking advantage of its special structure. Of course, in terms of computational efficiency, they are much better than general solvers.

The question is: are they applicable in each interaction of the approach here presented?

We tried to identify those cases where the introduction of two general extra constraints does not destroy completely the usefulness of specialized algorithms. At this moment we already identified some cases where the use of specialized algorithms remains clearly interesting and some cases where developing adaptations of the specialized algorithms seems possible. Clearly, in the first situation there is, for example, any problem where dynamic programming is a good approach. We were already able to place the simple plant location problem (SPLP) in the second situation. In fact, a successful adaptation of a very well known algorithm for the simple plant location problem, the DUALOC procedure of Erlenkotter (1978), was presented in Dias *et al.* (2003b).

The interactive process looks for a progressive and selective learning of the non-dominated solutions set, clarifying the criteria values aggregation meaning and consequences. Although in some situations it is possible to opt for one alternative in many others the interactive process just enables the elimination of great part of the feasible solutions reducing the final choice to a small part of the non-dominated ones. In this case, if necessary, these alternatives can be scrutinized using another multicriteria analysis tool dedicated to discrete problems, where the alternatives are known explicitly and in small number. Of course, this stage looks for a more detailed analysis of this sub-set of the non-dominated alternatives. In many cases it is justified to extend the analysis to slightly dominated solutions, in terms of the two objective function values considered in the first phase, very close to the previously selected solutions in the first phase. However, this second phase analysis does not enable the combinatorial nature of feasible solutions to be explored. So, it should only be used for a deeper study of alternatives filtered by the phase one of the process.

Once the preferred efficient solution or a set of compromise alternatives is known (some of them, eventually slightly dominated as referred to above, a second phase consisting in an *a posteriori* analysis procedure should be carried out, for a more detailed analysis of this sub-set of the alternatives. We are now developing some tools for this type of *a posteriori* analysis.

Acknowledgements

References

1. **Bouyssou, D.,** "Modelling inaccurate determination, uncertainty, imprecision using multiple criteria". In *Improving decision making in organizations,* edited by A .G. Lockett and G. Islei, pp. 78-87, Heidelberg, Springer Verlag, 1989.
2. **Cappanera, P., G. Gallo and F. Maffioli,** "Discrete Facility Location and Routing of Obnoxious Facilities", *Discrete Applied Mathematics*, 133: 3-28 (2004).
3. **Caruso, C., A. Colorni and M. Paruccini,** "The regional urban solid waste management system: A modelling approach", *European Journal of Operational Research*, 70: 16-30 (1993).
4. **Chalmet, L. G., L. Lemonidis and D. J. Elzinga,** "An algorithm for the bi-criterion integer programming problem", *European Journal of Operational Research*, 25: 292-300 (1986).
5. **Clímaco, J., C. H. Antunes and M. J. Alves,** *Programação Linear Multiobjectivo*, Coimbra, Imprensa da Universidade de Coimbra, Portugal (in portuguese), 2003.
6. **Clímaco, J., M. E. Captivo and C. Ferreira,** "Modelos discretos bicritério de programação matemática: nota sobre a aplicabilidade de uma aproximação interactive", *working paper* 8/97, Centro de Investigação Operacional, Faculdade de Ciências da Universidade de Lisboa (in portuguese), 1997.
7. **Costa, M. G., M. E. Captivo and J. Clímaco.,** "Capacitated Single Allocation Hub Location Problem - A Bicriteria Approach", *Computers & Operations Research*, 35 (11): 3671-3695 (2008).
8. **Coutinho-Rodrigues, J. M., J. Current, J. Clímaco and S. Ratick,** "Interactive spatial decision support system for multiobjective hazmat location-routing problems", *TRB* (A peer reviewed publication of the Transportation Research Board of the National Research Council, USA) record 1602: 101-109 (1997).
9. **Current, J., H. Min and D. Schilling,** "Multiobjective analysis of facility location decisions", *European Journal of Operational Research*, 49: 295-307 (1990).
10. **Dias, J., M. E. Captivo and J. Clímaco.,** "Desenvolvimento de um Sistema de Apoio à Decisão Dedicado ao Estudo de Problemas de Localização Dinâmica de Equipamentos para Transferência, Tratamento e Deposição de Resíduos Sólidos". In *Anais do XXXV SBPO*, Natal, Brasil, 906-922 (in portuguese), 2003a.
11. **Dias, J., M. E. Captivo and J. Clímaco,** "An Interactive Procedure dedicated to a Bicriteria Plant Location Problem", *Computers & Operations Research*, 30: 1977-2002 (2003b).
12. **Dias, J., M. E. Captivo and J. Clímaco.,** "Decision Support System for Location Problems". In *Proceedings of CIDMDS 2006* (ISBN 1-905800-00-2), edited by F. Adam, P. Brézillon, S. Carlsson and P. Humphreys, vol. 1, pp. 388-402, London, 2006.
13. **Erkut, E. and S. Neuman,** "Analytical models for locating undesirable facilities", *European Journal of Operational Research*, 40: 275-291 (1989).
14. **Erkut, E. and S. Neuman,** "A Multiobjective Model for Locating Undesirable Facilities", *Annals of Operations Research*, 40: 209-227 (1992).
15. **Erlenkotter, D.,** "A dual based procedure for uncapacitated facility location", *Operations Research*, 26: 992-1009 (1978).
16. **Fernandes, S., M. E. Captivo and J. Clímaco,** "SABILOC - Um Sistema de Apoio à Decisão para Análise de Problemas de Localização Bicritério", *Pesquisa Operacional*, 27 (3): 607-628 (in portuguese) (2007).
17. **Ferreira, C., J. Clímaco and J. Paixão,** "The location-covering problem: a bicriterion interactive approach", *Investigación Operativa*, 4: 119-139 (1994).
18. **Ferreira, C., B. S. Santos, M. E. Captivo, J. Clímaco and C. C. Silva,** "Multiobjective location of unwelcome or central facilities involving environmental aspects - a prototype of a decision support system", *JORBEL (Belgian Journal of Operations Research, Statistics and Computer Science)*, 36: 159-172 (1996).

19. **Ferreira, C., C. C. Silva, B. S. Santos, J. Clímaco, M. E. Captivo and J. P. Paixão,** "Bicriteria covering and related problems - a prototype of a decision support system". In Proceedings of the *5th Conference of CEMAPRE - Aplicações da Matemática à Economia e Gestão*, Lisboa, 415-428, 1997.

20. **Ferreira, C.,** Problemas de Localização e Distribuição Multicritério: aproximações e estudo de alguns casos com implicações ambientais, PhD Thesis, University of Aveiro, Portugal (in Portuguese), 1997.

21. **Figueira, J.,** L'Approche interactive dans le cas des problémes de flot multicritéres, Thèse de Doctorat, Université Paris-Dauphine, France, 1996.

22. **Fonseca, M.C. and M. E. Captivo,** "Location of semi obnoxious facilities with capacity constraints", *Studies in Locational Analysis*, 9: 51-52 (1996).

23. **Fonseca, M.C. and M. E. Captivo,** "Models for Semi obnoxious Facility Location", *Proceedings of the XIII CLAIO*, Montevideo, Uruguay (2006).

24. **Fonseca, M.C. and M. E. Captivo.,** "Analysis of Bicriteria Semi obnoxious Location Models". Paper presented at *Tercer Encuentro de la Red Iberoamericana de Evaluación y Decisión Multicriterio, R.E.D.-M. 2007,* Culiacán, México, 2007.

25. **Giannikos, I.,** "A Multiobjective Programming Model for Locating Treatment Sites and Routing Hazardous Wastes", *European Journal of Operational Research*, 104: 333-342 (1998).

26. **Haastrup, P., V. Maniezzo, M. Mattarelli, F. Rinaldi, I. Mendes and M. Paruccini,** "A Decision Support System for Urban Waste Management", *European Journal of Operational Research*, 109 (2): 330-341 (1998).

27. **Hultz, J., D. Klingman, G. T. Ross and R. Soland,** "An Interactive Computer System for Multicriteria Facility Location", *Computers and Operations Research*, 8 (4): 249-261 (1981).

28. **Lahdelma, R., P. Salminen and J. Hokkanen,** "Locating a Waste Treatment Facility by Using Stochastic Multicriteria Acceptability Analysis with Ordinal Criteria", *European Journal of Operational Research*, 142 (2): 345 356 (2002).

29. **Larichev, O. and A. Nikiforov,** "Analytical survey of procedures for solving multicriteria mathematical programming problems", in *Towards interactive and intelligent decision support systems* 1, pp. 95-104, Springer-Verlag, 1987.

30. **Malczewski, J. and W. Ogryczak,** "An interactive approach to the central facility location problem: locating pediatric hospitals in Warsaw", *Geographical Analysis*, 22: 244-258 (1990).

31. **Melachrinoudis, E., H. Min and X. Wu,** "A Multiobjective Model for the Dynamic Location of Landfills", Location Science, 3: 143-166 (1995).

32. **Ogryczak, W., K. Studzinski and K. Zorychta,** "DINAS – Dynamic interactive network analysis system". In *Aspiration based decision support systems* edited by A. Lewandowski, A. P. Wierzbicki, pp. 385-387, *Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, Berlin 331, 1989.

33. **Rakas, J., D. Teodorovic and T. Kim,** "Multi-objective Modeling for Determining Location of Undesirable Facilities", *Transportation Research Part D,* 9: 125-138 (2004).

34. **Revelle, C. and G. Laporte,** "The Plant Location Problem: New Models and Research Prospects", *Operations Research*, 44 (6): 864-873 (1996).

35. **Ross, G. and R. Soland,** "A multicriteria approach to the location of public facilities", *European Journal of Operational Research*, 4: 307-321 (1980).

36. **Wierzbicki, A. P.,** "A Mathematical basis for satisficing decision making", *Mathematical Modelling*, 3: 391-405 (1982).

37. **Wyman, M. and M. Kuby,** "A Multiobjective Location-Allocation Model for Assessing Toxic Waste Processing Technologies", Studies in *Locational Analysis*, 4: 193-196 (1993).

38. **Wyman, M. and M. Kuby,** "Proactive Optimization of Toxic Waste Transportation, Location and Technology", *Location Science*, 3 (3): 167-185 (1995).

39. **Zeleny, M.,** Linear multiobjective programming, *Lecture Notes in Economics and Mathematical Systems*, Berlin, Springer-Verlag, 95, 1974.

Maria Eugénia Captivo *obtained her Ph.D. in Operations Research in 1988 and the title of "Agregação" 1n 2005 both at the University of Lisbon. She is Associate Professor of Operations Research at the Faculty of Science, University of Lisbon, and a researcher at the Operations Research Center (member of the executive committee from 1994 to July 2003). She was Vice-President of APDIO - the Portuguese OR Society (1990-94). She is author/co-author of over 50 scientific articles in journals, books and proceedings (all with referees) in combinatorial optimization, mostly in the areas of location, network optimization and paper production planning and cutting. She has supervised 8 PhD and 14 MSc students. She participated in several research and development projects. Her current research interests are in locational analysis, combinatorial optimisation, network optimisation, production planning, cutting and packing.*

João Carlos Namorado Clímaco *is Full Professor at the Faculty of Economics of the University of Coimbra and President of the Scientific Committee of INESC - Coimbra. He obtained the M.Sc. Degree in Control Systems at the Imperial College of Science and Technology, University of London (1978); the Ph.D. in Optimization and Systems Theory (1982) and the title of "Agregação" (1989) at the University of Coimbra. He was, in the past, Vice-President of ALIO - Latin Ibero American OR Association and of APDIO - the Portuguese OR Society and Member of the International Executive Committee of the International Society on Multiple Criteria Decision Making. Now he is member of the IFIP WG 8.3 on Decision Support Systems. He belongs to the Editorial Board of Journal of Group Decision and Negotiation, International Transactions in Operational Research (ITOR), "Investigação Operacional" (Journal of the Portuguese OR Society), and ENGEVISTA (a Brazilian Journal) and of the University of Coimbra Press. He is author/co-author of more than 140 papers using the peer refereeing system. His current major interests of research are: multiple criteria decision aiding, multiobjective combinatorial problems, and management and planning of telecommunication networks.*