Modeling and Control in Task-Space of a Mobile Manipulator with Cancellation of Factory-Installed Proportional–Derivative Control

Gastón H. Salazar-Silva[1], Marco A. Moreno-Armendáriz[2], and Jaime Álvarez Gallegos[3]

[1]Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas,
Instituto Politécnico Nacional,
Mexico City,
Mexico

[2]Centro de Investigación en Computación, Instituto Politécnico Nacional,
Mexico City,
Mexico

[3]Secretaría de Investigación y Posgrado, Instituto Politécnico Nacional,
Mexico City,
Mexico

ghsalazar@ipn.mx

Abstract. A mobile manipulator is a robotic arm mounted on a mobile robot; a particular example is a manipulator arm on a mobile robot with differential traction. Mobile manipulators have many advantages over stationary manipulator, such as a larger work space than a stationary manipulator could have in practice. This paper shows a systematic approach to modeling mobile manipulators that transforms the problem to the modeling of a stationary manipulator with non-holonomic kinematic constraints on the joints. It is also presented a task-space control that cancels a factory-installed proportional–derivative (PD) control and it uses an estimate of the derivative of the posture kinematic model. Finally, a numerical experiment is presented using this method.

Keywords. Robot control, mobile manipulators, robots kinematics.

Modelado y control en espacio de tarea de un manipulador móvil con cancelación de control proporcional–derivativo instalado en fábrica

Resumen. Un manipulador móvil es un sistema compuesto por un manipulador estacionario montado sobre un robot móvil; un ejemplo particular es un brazo manipulador montado sobre un robot de tracción diferencial. Los manipuladores móviles presentan varias ventajas con respecto a manipuladores estacionarios, por ejemplo un mayor espacio de trabajo. En el presente trabajo se muestra un método para el modelado de manipuladores móvil que transforma el problema a el modelado de un manipulador estacionario con restricciones cinemáticas no holónomas en las articulaciones. También se presenta un control en el espacio de tarea que cancela un control proporcional–diferencial proveniente de fabrica y usa un estimado de la derivada del modelo cinemático de postura. Finalmente, se presentan los resultados obtenidos en un experimento numérico.

Palabras clave. Control de robot, manipuladores móviles, cinemática de robots.

1 Introduction

The robots are coming out from the structured environments in factories and they begin to appear in places such as houses, offices and hospitals, where the surroundings have little structure or not at all [8]; the mobile manipulators are a solution for these new work spaces. Basically, a mobile manipulator is a stationary manipulator mounted on a mobile robot so the locomotion and manipulation tasks may be performed simultaneously (Figure 1); these capabilities give to the mobile manipulator advantages over stationary manipulators like a bigger task space and a greater autonomy; a mobile manipulator also has disadvantages, such as non-holonomic kinematic constraints. Previously, the mobile-manipulator control was focused in handling separately the tasks of locomotion and manipulation, for example in [17] the locomotion problem is the issue, or in [7, 19] where the manipulation was the control problem; now these two tasks can be handled as one problem; for example, in [2, 17], a kinematic control is developed; there are literature that presents results with the dynamic control of a mobile manipulator; for example in [1] a dynamic control is used to reduce the balancing oscillation; also, a dynamic redundancy resolution control is applied in [18] but the control is not robust; in [9] a optimal dynamic control is developed to track a trajectory considering the maximum load-carrying capacity of the robot while it avoids obstacles.

On the other hand, the kinematic modeling problem is still handled as two separate problems, and the resulting models are then combined to obtain a model of the whole robot; these problem are solved using different techniques, for example in [4, 10, 15, 14, 13, 3]; in [12] a method is presented that combines the kinematic models of the mobile base and the manipulator, but the mobile base and the manipulator are still modeled with different methods; very interesting examples are [14] and [13], where both the mobile base and the manipulator have non-holonomic constraints and yet are modeled by different methods. A fundamental work for mobile-robots modeling is [5]; in it a classification for wheeled mobile robots is presented, based on their degree of mobility and degree of maneuverability, and four types of models are proposed.

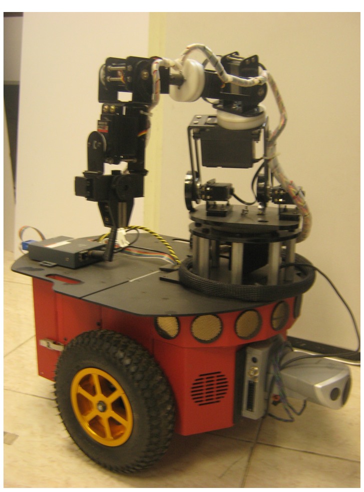

Figure 1. The experimental system is composed of a by a Pioneer 3DX mobile robot and a Cyton manipulator arm with 7 DOF

The present paper shows a new methodology for modeling and control a mobile manipulator assuming that the robot behaves as an stationary manipulator with kinematic constraints on the joints. The outline of this work is as follows: First a review on modeling techniques for stationary manipulators and mobile robots is presented (Section 2). After that, an integrated modeling technique for kinematic models of mobile manipulators is presented (Section 3) and applied to obtain a dynamic model of the mobile manipulator (Section 4). To obtain a kinematic model the non-holonomic constraints are modeled as a mapping between the actuation space and the joint space, using the so called configuration kinematic model. Then, with the obtained kinematic and dynamic models, a task-space control is developed (Section 5). Finally, the results of numerical simulations for the task-space control are showed assuming a 5-DOF differential-traction mobile manipulator (Section 6).

2 Kinematic Modeling Techniques for Stationary and Mobile Robots

In a stationary manipulator, a kinematic model describes the relation between the motion of a mechanical system and the motion of the actuators. This model is conceptually obtained through the so called forward kinematics, which is the function that describes the relation between the posture of the final effector of the robot in task space and the value of the joint variables of the robot. The forward kinematics is usually expressed as

$$r_m = f_m(q) \tag{1}$$

where $r_m \in \mathbb{R}^p$ is the posture variables vector, $q \in \mathbb{R}^n$ is the joints displacement vector, and n is the dimension of q, usually called degree of freedom (DOF). A widely used tool to obtain the forward kinematics of a stationary manipulator is the homogeneous transformation matrix [16]; this matrix can be constructed with the help of the Denavit–Hartenberg parameters; these parameters describes the geometric characteristics of the links and joints of a robot. The kinematic model of a stationary manipulator with full-actuated independent joints is then obtained through the time derivative of (1)

$$\dot{r}_m(t) = J_m(q)\dot{q}(t) \tag{2}$$

where $\dot{r}_m \in \mathbb{R}^p$ are the posture velocities, $\dot{q} \in \mathbb{R}^n$ are the joint velocities of the manipulator, and the matrix $J_m(q) \in \mathbb{R}^{p\times n}$ is the so called Jacobian and it is defined as

$$J_m(q) = \frac{\partial f_m}{\partial q}(q).$$

Another way to compute the Jacobian is the geometric method [16], that has the advantage of not requiring an explicit computation of the derivatives of the forward kinematic, as uses numeric information from the homogeneous transformations.

On the other hand, the motion of a wheeled mobile robot is characterized by the kinematic constraints imposed by the wheels [11, 5]. A kinematic constraint may be holonomic or non-holonomic. A mechanical system is said to be *holonomic* if there is a set of k constraints to the motion; these constraints may be expressed as

$$h_i(q) = 0,\ i = 1,\ldots,k$$

where h_i are scalar functions; such constraints are geometric and they limit where may be the system configuration. If the mechanical system is limited by constraints expressed as

$$a_i(q,\dot{q}) = 0$$

then the system is called *non-holonomic* and these constraints limit how the mechanical system can move, but not restrict where the system can be. A special kind of non-holonomic constraint is the so the called *Pfaffian* form, in where the constraint equations are linear with respect the joint velocities

$$a_i(q)\dot{q} = 0.$$

The kinematic model of a mobile robot can be obtained from the null space of the non-holonomic kinematic constraints of the robot.

In mobile robots, there are two kinds of kinematic models [11, 5]. The first model establishes the relation between the motion of the final-effector posture and the actuators motion. This model is called posture kinematic model and it may be used to produce the dynamic model of a mobile robot. This model is similar to kinematic model of a stationary manipulator. The posture of a wheeled mobile robot on a flat surface is completely specified by the vector

$$r_b := \begin{pmatrix} x \\ y \\ \phi \end{pmatrix} \tag{3}$$

where x and y are the coordinates on the plane on which the mobile robot moves and ϕ is the robot orientation on such plane. If the mobile robot has centered orientable wheels, the posture must be extended to include the angles of the wheels. The posture kinematic model of a wheeled mobile robot with differential traction can be expressed as [5]

$$\dot{r}_b(t) = B(q)\eta_b \tag{4}$$

where $\eta_b(t) \in R^{n-k}$ is the vector which contains the velocities of the actuators, and $B(q_b) \in R^{n\times(n-k)}$ is a matrix with its columns being a base of the null space of the non-holonomic constraints; The posture kinematic model is useful in the computation of control laws in the task space; on the other hand, the configuration kinematic model is used to simplify the dynamic model of mobile robot.

The second model describes a relation between the joints motion and the actuators motion, called configuration kinematic model, and it is defined as

$$\dot{q}_b = S_b(q)\eta_b \tag{5}$$

where $S_b(q) \in R^{n\times(n-k)}$ is a matrix with its columns belongs to the null space of the constraint matrix A_b. For example, in a differential-traction mobile robot, the configuration kinematic model can be defined as

$$S_b(q) = \begin{pmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{pmatrix}.$$

It is important to remark that $S_b(q)$ is an annihilator of the kinematic constraints, such that

$$A_b(q)^T S_b(q) = 0 \tag{6}$$

where the matrix $A_b(q) \in \mathbb{R}^{n\times k}$ defines the non-holonomic kinematic constraint

$$A_b(q)\dot{q} = 0.$$

This fact is used to simplify the dynamic model.

It is important to remark that the expressions (5) and (4) describe a system motion in terms of the actuators motion; in these models, there is no information about the forces that generate the motion, so they do not describe the dynamics of the system; however, in some applications, it is enough to have a kinematic model for the development of a control law. Also, the wheeled-mobile-robot kinematic model is useful to obtain the robot dynamic model; for example, the posture kinematic model can be used to obtain the dynamic model of the robot; on the other hand, in a non-holonomic wheeled mobile robot the kinematic constraints appear on the dynamic model and the configuration kinematic model is used to eliminate these constraints [11].

3 Kinematic Modeling of Mobile Manipulators

In this section, a method is presented which assumes that the mobile manipulator is a stationary manipulator with non-holonomic kinematic constraints on the robot joint.

As stated before, the kinematic models of mobile manipulators are developed by separately obtaining the kinematic models of the mobile base and the mounted manipulator, and the combining both models. One of such methods uses the so called extended Jacobian [12], which is defined as

$$\dot{r} = \begin{pmatrix} J_b(q)S_b(q) & J_m(q) \end{pmatrix} \eta \tag{7}$$

where J_b is the Jacobian of the base, which is obtained directly from the time derivative of (3), $\dot{r}$ is a vector that combines the posture motion of the mobile base and the manipulator arm

$$\dot{r} = \begin{pmatrix} \dot{r}_b \\ \dot{r}_m \end{pmatrix},$$

and $\eta \in \mathbb{R}^m$ are the actuators velocities for the mobile manipulator and are defined as

$$\eta = \begin{pmatrix} \eta_b \\ \dot{q}_m \end{pmatrix}$$

where $\dot{q}_m$ are the velocities vector of the mounted manipulator and wich also correspond to the velocities of the manipulator-arm actuators.

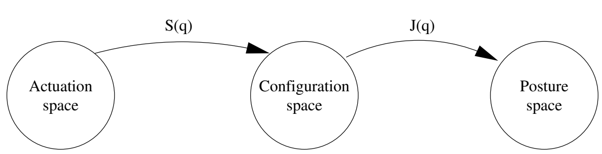

Figure 2. The posture kinematic model as a composition of mappings between the actuation space, the configuration space and task space

The configuration kinematic model is a mapping between the actuation space, which is the set of all possible motions of the actuators, and the joint space (Figure 2); these motions are restricted by the kinematics constraints and the mapping is usually an identity matrix in a stationary manipulator. The Jacobian is a mapping between the joint space and the posture space; the motions described by the Jacobian are not restricted by the kinematic constraint. Finally, the posture kinematic model is the combination of the Jacobian and the configuration kinematic model. Thus, the kinematic modeling of a mobile manipulator depends on finding the Jacobian J and this in turn depends on combining the kinematics of the manipulator and the base mobile. The kinematics of a mobile manipulator is given by the function f, defined as

$$r = f(q) \tag{8}$$

where r is the combined posture of the mobile manipulator and q are the generalized coordinates of the mobile manipulator, defined as

$$q = \begin{pmatrix} q_b \\ q_m \end{pmatrix}$$

where q_b and q_m are the generalized coordinates of the mobile base and the manipulator arm respectively. It is important to note that the function f is not subject to the kinematic constraints.

A method to find the direct kinematics of the manipulator arm and mobile base, and also allows combine them, are the homogeneous transformations [16]; specifically for the mobile manipulator it is defined as [10]:

$$T_n^0 = T_b^0 T_n^b$$

where T_b^0 is the homogeneous transformation which goes from a frame $\{b\}$ fixed on the mobile base to a frame $\{0\}$ fixed in the surface on which the mobile base moves, and T_n^b is the homogeneous transformation which goes from a frame $\{n\}$ fixed on the last link of the mobile manipulator to the frame $\{b\}$. In the reviewed literature there is not a standardized method to find the transformation T_b^0; a possible method is modeling the mobile base as a solid body. It is important to remark that the forward kinematics does not take account of the non-holonomic constraints.

The proposed method consists in obtaining the forward kinematics of the mobile base T_b^0, by initially assuming that the mobile base is stationary manipulator of b DOF; for example, a differential traction mobile robot could be modeled as link connected to the surface with planar joint, which in turn could be modeled as two prismatic joint and a revolute joint; as a general case, every mobile robot, for example a flying vehicle, could be modeled as an equivalent 6 DOF stationary manipulator. Following this assumption, it is possible to obtain the forward kinematics of the whole mobile manipulator by considering it a unique kinematic chain, and the applying a standard modeling method for stationary robots, such as the Denavit–Hartenberg method. Also, following the last assumption, the same geometric method used to obtain the Jacobian J_b in stationary robots can be applied on mobile manipulators and, further, find the Jacobian of the whole mobile manipulator

$$J(q) = \begin{pmatrix} J_b(q) & J_m(q) \end{pmatrix}$$

with the geometric method, and (7) is reduced to the posture kinematic model of mobile manipulator

$$\dot{r} = B(q)\eta \tag{9}$$

where $B(q)$ is the posture kinematic relation of the mobile manipulator, defined as

$$B(q) = J(q)S(q)$$

$S(q)$ is the configuration kinematic relation for the whole mobile manipulator

$$S(q) = \begin{pmatrix} S_b(q) & I \end{pmatrix}$$

where I is a identity matrix, that indicates which configuration velocities are identical to actuation velocities.

The proposed methodology has many advantages, for example it uses the same methods as the stationary manipulators to obtain the forward kinematics and the kinematic models. Another advantage is that the existing computational tools for stationary robots could be used, for example [6]; then it is possible to obtain numerically the dynamic model.

4 Dynamic Model of a Mobile Manipulator with Proportional–Derivative Control

The dynamic model of a mechanical system with non-holonomic constraints is defined by a set of n second-order differential equations [11]

$$\begin{array}{rcl} D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) & = & A(q)\lambda + S(q)\tau \\ A(q)^T\dot{q} & = & 0 \end{array} \tag{10}$$

where $D(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix for the system, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis and cross-velocities matrix, $g(q) \in \mathbb{R}^n$ is a vector which represents the impact of gravity on the links, $A(q) \in \mathbb{R}^{n\times k}$ is a matrix in which a set of k kinematic constrains are expressed, $S(q) \in \mathbb{R}^{n\times m}$ in the input matrix, and $\tau \in \mathbb{R}^m$ are the generalized forces that go into system, which is defined as

$$\tau = \begin{pmatrix} \tau_b \\ \tau_m \end{pmatrix} \tag{11}$$

where τ_b and τ_m are the generalized forces on the mobile base and the manipulator respectively.

Taking advantage of the relation (6), the explicit statement of the kinematic constraints in (10) could be eliminated; in order to achieve this, the equation (10) is pre-multiplied by $S(q)^T$ and it is then the transformation (4) is applied, thus the following expression is obtained [11]

$$\begin{array}{rcl} \dot{q} & = & S(q)\eta \\ \dot{\eta} & = & -M(q)^{-1}m(q,\eta) \\ & & +M(q)^{-1}S(q)^T S(q)\tau \end{array} \tag{12}$$

where

$$\begin{array}{rcl} M(q) & = & S(q)^T D(q) S(q) \\ m(q,\eta) & = & S(q)^T D(q)\dot{S}(q)\eta \\ & & +S(q)^T C(q,S(q)\eta)S(q)\eta \\ & & +S(q)^T g(q) \end{array}.$$

Remark: there is a dimension reduction of the state in (10) compared with (12) due to the kinematic constraints.

In an off-the-shelf robot, it is usual that comes with an installed control, such as a PD control. In this section,

the assumption is made that the mobile base has a PD control, which has the form

$$\tau_b = K_1\left(\eta_b^d - \eta_b\right) + K_2\left(\dot{\eta}_b^d - \dot{\eta}_b\right) \tag{13}$$

with proportional and derivative gain matrices K_1 and K_2, which are programmable; and the manipulator has a PD regulator in the form

$$\tau_m = K_3\left(q_m^d - q_m\right) - K_4\,\dot{q}_m \tag{14}$$

where the matrices K_3 and K_4 are proportional and derivative gains. Then (13) and (14) are applied to (12) and with the assumption that K_2 is identical to zero, the result is a system that can be expressed as

$$\begin{aligned} \dot{q} &= S(q)\eta \\ \dot{\eta} &= -M(q)^{-1}m(q,\eta) \\ &\quad +M(q)^{-1}S(q)^T S(q)Ku \\ &\quad -M(q)^{-1}S(q)^T S(q)\left(K_p q + K_d\eta\right) \end{aligned} \tag{15}$$

where $u \in \mathbb{R}^m$ is the reference input to the system, K is a full rank matrix which is defined as

$$K = \begin{pmatrix} K_1 & O \\ O & K_3 \end{pmatrix}, \tag{16}$$

K_p is expressed as

$$K_p = \begin{pmatrix} O & O \\ O & K_3 \end{pmatrix}, \tag{17}$$

and K_d is defined as

$$K_d = \begin{pmatrix} K_1 & O \\ O & K_4 \end{pmatrix}. \tag{18}$$

5 Control Design for the Mobile Manipulator

In this section, a classical combination of two control loops in cascade is proposed [16]; the internal loop control uses an inverse dynamics compensator with PD cancellation; this will be analyzed first. Then an external control loop will be presente which is a robust resolution of acceleration control over the task space.

5.1 Inverse Dynamic Compensator with PD Cancellation

As previously established, the mobile manipulator already has a PD controls. Then the internal control loop may be defined through the inverse dynamics of the mobile manipulator with a compensator to cancel the factory-installed PD control. The inverse dynamics compensator uses the expression (15) to find a suitable τ such that cancels the dynamics of the system and the PD control that was previously applied

$$u = (S(q)^T S(q))^{-1} K^{\dagger}\left(M(q)a + m(q,\eta) + K_p q + K_d v\right) \tag{19}$$

where $a(t) \in \mathbb{R}^4$ is the acceleration reference for the system. Applying (19) to (15) results in the linear system

$$\begin{aligned} \dot{q} &= S(q)\eta \\ \dot{\eta} &= a; \end{aligned} \tag{20}$$

the new system (20) can have any other desired control in an external loop.

5.2 Task-Space Control

For the external control loop, the resolution of acceleration control (RAC) is used. First, a measure of the error on task space is proposed, $\tilde{r}$, such that

$$\tilde{r}(t) = r^d(t) - r(t).$$

where $r^d(t) \in \mathbb{R}^n$ is the desired posture. Then the control is proposed according to the following error dynamics

$$\ddot{\tilde{r}}(t) + K_1\dot{\tilde{r}}(t) + K_0\tilde{r}(t) = 0 \tag{21}$$

where $\dot{\tilde{r}}$ and $\ddot{\tilde{r}}$ are the first and second derivatives of the error with respect time.

From (21), the following control law is obtained

$$\ddot{r}(t) = \ddot{r}^d(t) + K_1\dot{\tilde{r}}(t) + K_0\tilde{r}(t), \tag{22}$$

which is a task-space PD control with the desired posture acceleration as feed-forward. However, the actuators of the mobile manipulator are not defined in task space, so it is required to transform the control (22) to the actuation space.

To resolve the required acceleration on the actuators, the time derivative of (5) is used

$$\ddot{r} = \dot{B}(q)\eta + B(q)\dot{\eta}. \tag{23}$$

Then (22) in combination with (5) are used to obtain

$$\dot{\eta} = B(q)^{\dagger}\left(\ddot{r}^d - \dot{B}(q,\eta)\dot{\eta} + K_1\dot{\tilde{r}} + K_0\tilde{r}\right). \tag{24}$$

A problem with the control (24) is that it needs the product $\dot{B}(q)\eta$; in the present section a method is proposed to estimate numerically the value of such expression. This method uses only numerical information about the values of B does not require its derivative, which can complex to obtain. First, it is required to solve (23) for $\dot{B}\eta$

$$\dot{B}(q)\eta = \ddot{r} - B(q)\dot{\eta}; \tag{25}$$

Then, definitions of the derivatives $\ddot{r}$ and $\dot{\eta}$ are applied

$$\dot{B}(q)\eta = \frac{d}{dt}\dot{r} - B(q)\frac{d}{dt}\eta$$

and finally the expression (9) is used to replace the first term on the right

$$\dot{B}(q)\eta = \frac{d}{dt}(B(q)\eta) - B(q)\frac{d}{dt}\eta. \tag{26}$$

The expression (26) can be simply approximated as the derivatives of vector signals.

6 Numerical Experiments and Results

To test the proposed method, a mobile manipulator was modeled; it is integrated by a Pioneer 3DX mobile robot and a Cyton manipulator arm with 7 DOF. The Pioneer 3DX is a differential traction mobile robot and only two joints of the Cyton robot were considered, thus the mobile manipulator is modeled as a 5 DOF system; the other joints of the Cyton robot where considered fixed and were not modeled.

To obtain the kinematic constrains it is assumed that the mobile manipulator is a unicycle without slipping; also the surface on which the mobile base moves is flat and horizontal. It is also assumed that the manipulator arm is a 2-joint planar robot, and its links are modeled like rods.

The mobile manipulator was modeled with help of the computational algebra system `Maxima`, version 5.21.0 using Lisp SBCL 1.0.29.11. The model was also obtained numerically using the Matlab's `robotics toolbox` [6]; this toolbox is used for modeling stationary robot but it can be applied to the modeling of mobile manipulator.

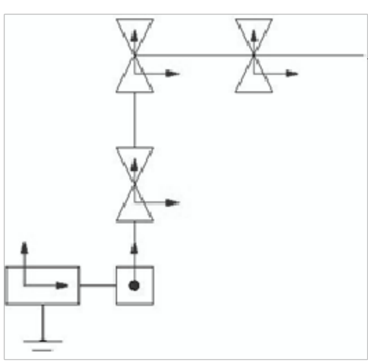

Figure 3. Kinematic representation of mobile manipulator 5 DOF as a stationary manipulator for the purpose of modeling. The first three joints represent the mobile base

To obtain the forward kinematics, the mobile base is modeled as a 2-joint Cartesian manipulator and a third revolute joint, as appears on Figure 3. From this description the Denavit–Hartenberg parameters can be obtained (Table 1).

Table 1. The Denavit–Hartenberg parameters for the 5-DOF mobile manipulator. The angles are in radians and the distances in milimeters

i	α	a [mm]	θ	d [mm]	Kinematic pair
1	$-\pi/2$	0	0	0	prismatic
2	$\pi/2$	0	$-\pi/2$	0	prismatic
3	0	0	0	237	revolute
4	0	150	0	0	revolute
5	0	168	0	0	revolute

6.1 Kinematic Model

Following the assumption that the mobile manipulator could be modeled as a stationary manipulator, as shown in Figure 3, the configuration of the mobile manipulator, $q(t) \in \mathbb{R}^5$, is defined as:

$$q = \begin{pmatrix} d_1 \\ d_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{pmatrix}$$

where d_1, d_2 are the surface coordinates (x,y) of the mobile base, $\theta_3 = \phi$ is the orientation of the mobile base, and θ_4, θ_5 are the joint variables of the manipulator arm.

On the other hand, the kinematic constraint of the 5-DOF mobile manipulator is given by the matrix $A(q) \in \mathbb{R}^{5\times 1}$ and it is defined by the expresion

$$A(q) = \begin{pmatrix} \sin q_3 \\ -\cos q_3 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{27}$$

A possible configuration kinematic model that satisfy is the equation

$$\dot{q} = S(q)\eta \tag{28}$$

where $\eta \in \mathbb{R}^4$ are actuation velocities, defined as:

$$\eta = \begin{pmatrix} v \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{pmatrix}$$

where $v(t)$ is an scalar which describes the linear velocity of the mobile robot and the configuration kinematic model $S(q) \in \mathbb{R}^{5\times 4}$ is defined by

$$S(q) = \begin{pmatrix} \cos q_3 & 0 & 0 & 0 \\ \sin q_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{29}$$

which satisfy the property of being an annihilator for (6.1).

6.2 Dynamic Model

The matrices $D(q)$ and $C(q,\dot{q})$ of the system (10) are obtained from the procedure presented in [16]. To calculate those matrices, some data about the robot link are needed; these data appears in Table 2; an important remark is that the links 1 and 2 are considered massless and therefore do not affect the dynamics.

Table 2. Link data from the mobile manipulator

i	Length [mm]	Wide [mm]	Height [mm]	Mass [kg]
3	445	393	237	9.0
4	150	50	50	0.1
5	168	50	50	0.1

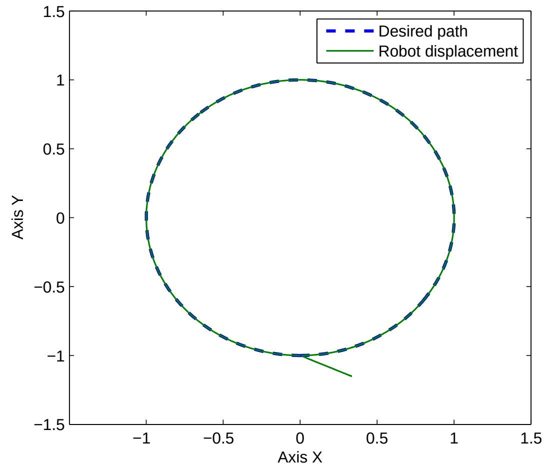

Figure 4. The reference path and the motion of the robot

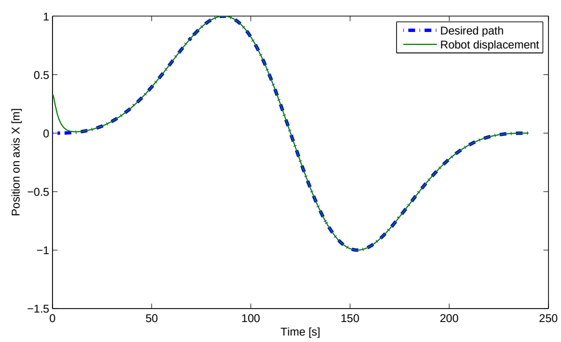

Figure 5. Trajectory tracking on axis X

6.3 Results

The control described in the Section 5 was applied to a numerical model of the mobile manipulator. The reference is a circular trajectory in task space (Figure 4) and it is generated by the equations

$$\begin{aligned} x(t) &= \cos(\alpha(t)) \\ y(t) &= \sin(\alpha(t)). \end{aligned} \tag{30}$$

where α is a function of time define by a third order polynomial; the velocities and accelerations are define by the first and second time derivatives of (30). It can be observed that the robot follows very well the proposed trajectory (Figure 4, 5 and 6). An important remark is that the motion is counterclockwise and the initial movements of the robot are in the other direction.

The tracking error converges exponentially to zero and it is stable in the time frame of the simulation (Figure 7). On the other hand, the displacements of the joints are bounded (Figure 8), and it is noted that the second joint of the manipulator does not move and it is because the mobile manipulator is redundant.

In order to establish the robustness of the control, another two sets of experiments were executed. The objective of the first set of experiments was to measure the behavior of the pose error if there are parameters variations; in order to accomplish this objective, the experiments were modeled using a 2-level factorial experiment design for the following group of parameters: the mobile-base height, d_3, the mass of the mobile base, m_3, the lengths of the first link and the second link length of the manipulator, a_4 and a_5; sixteen numeric experiments were executed, where fourteen experiments were successful and

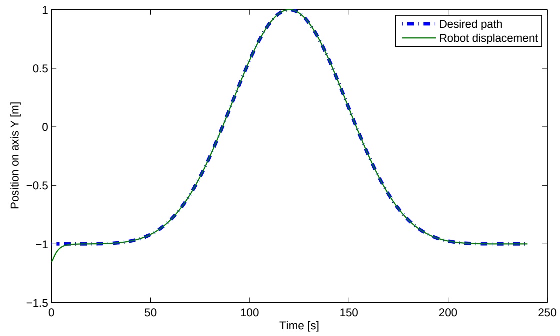

Figure 6. Trajectory tracking on axis Y

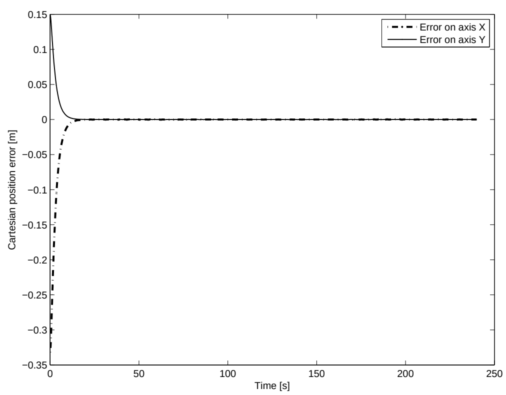

Figure 7. Posture error graph for the mobile manipulator under the control

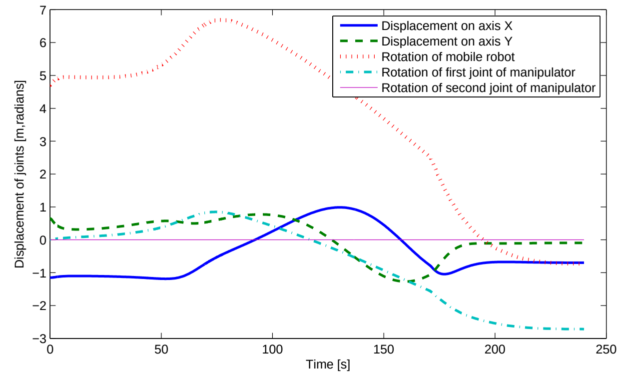

Figure 8. Displacements of the joints

Table 3. RMS error of the numeric–experiments results for variations in parameters, noise in measurements and no variations nor noise

Experiment set	Mean	Std. Dev. of the sample
Variation in parameters	0.569	0.087
Noise in measurements	0.627	0.020
No variations nor noise	0.589	Not Apply

in two experiments the system was numerically rigid as indicated by the numerical plataform. The objective of the second set of experiments was to measure the behavior of the error when there is noise on the pose measurements; a standard error of 0.05% was assumed with an interval of confidence of 97%; sixteen experiments were performed, where thirteen experiment were successful and three experiments were numerically rigid. On the obtained data of each experiment the root mean square (RMS) error was calculated, then the average of RMS error and the standard deviation of the sample was calculated for each set of experiments (Table 3). The results of the experiments show that the system of mobile manipulator and control is robust.

7 Conclusions

This paper shows a systematic approach to modeling mobile manipulators that transforms the problem to the modeling of a stationary manipulator stationary with non-holonomic kinematic constraints on the joints. Also, a task-space control was presented that consist in an internal compensator of the dynamics of the mobile manipulator and the factory installed PD control, and an external PD control with feed-forward of the posture acceleration and an estimate of the derivative of the posture kinematic model. Finally, three sets of numerical experiment are presented using the proposed control and the results show that the control is robust.

In future work, it will develop a robust priority control in the task space for a mobile manipulator. Also, it will be developed a robot-aided manipulation system to test these controls.

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Gastón H. Salazar-Silva received the B.S.E.E. from the Instituto Tecnológico de Mexicali, Mexicali B.C., México in 1993 and the M.Sc. In Automatic Control from the Centro de Investigaciones Avanzadas del Instituto Politécnico Nacional (CINVESTAV-IPN), Mexico City, in 2002. From 1999 he is professor at the Unidad Profesional Interdisciplinaria en Biotecnología del IPN. His research interests includes Robotics, Artificial Intelligence and Machine Vision.

Marco A. Moreno-Armendáriz received the B.S. degree from La Salle University, Mexico in 1998 and the M.S. and Ph.D. degrees, both in Automatic Control, from CINVESTAV-IPN, México, in 1999 and 2003, respectively. From 2001 to 2006 he was chair professor at the Engineering School in La Salle University, Mexico. In April of 2006, he joined the Center for Research in Computing of the National Polytechnic Institute, Mexico, where he is currently the head of the Computer Science Department. His research interests include Neural Networks for identification and control, Computer Vision, Robotics, Pattern Recognition and the implementation in FPGAs of related algorithms.

Jaime Alvarez-Gallegos was born in Tampico, Mexico. He received the B.S. Degree in electronics engineering from the National Polytechnic Institute (IPN), Mexico City, Mexico, in 1973 and the M.Sc. and Ph.D. degrees in electrical engineering from the Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional (CINVESTAV), in 1974 and 1978, respectively. He was the Head of the Department of Electrical Engineering, CINVESTAV (1992–1996), the Director of the School of Interdisciplinary Engineering and Advanced Technologies at IPN (1997–2000) and the Director of the Computing Research Center at IPN (2007–2009). He is currently the Head of the Secretary of Research and Graduate Studies at IPN. He was a Visiting Professor at the Imperial College of Science and Technology, London, U.K., in 1985–1986. His fields of interest are mechatronics, power electronics, optimization methods, and nonlinear control systems.

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