Novel Two-Stage Comb Decimator

Gordana Jovanovic Dolecek[1] and Sanjit K.Mitra[2]

[1]National Institute for Astrophysics, Optics and Electronics, Department of Electronics
INAOE, Puebla, Puebla
Mexico

[2]University of California, Department of Electrical and Computer Engineering
Santa Barbara, California
USA

gordana@inaoep.mx, mitra@ece.ucsb.edu

Abstract. A simple method for the design of a multiplier-less comb-based decimation filter is presented. The filter compensates the comb passband droop, and has the desired attenuation in the first folding band. The proposed structure has two decimation stages, both with simpler comb filters. The desired attenuation in the first folding band is achieved by an appropriate number of cascaded combs in the second stage. A simple compensation filter, working at a low output rate, provides the desired compensation in the pass band. The choice of the design parameters is presented along with a comparison of the proposed method to some other existing design methods.

Keyword. Comb filter, decimation, folding bands, compensation.

Nuevo decimador comb de dos etapas

Resumen. Se presenta un método simple para el diseño de un filtro de diezmado basado en filtro comb sin multiplicadores. El filtro compensa la caída en la banda de paso del filtro comb y tiene la atenuación deseada en la primera banda del plegado. La estructura propuesta tiene dos etapas de decimación, ambas con filtros comb simples. La atenuación deseada en la primera banda del plegado se alcanza por un número apropiado de combs en cascada en la segunda etapa. Un filtro simple de compensación trabajando en la baja razón de muestreo de salida provee la compensación deseada en la banda de paso. La elección de los parámetros de diseño es presentada junto con una comparación del método propuesto con algunos otros métodos de diseño existentes.

Palabras Clave. Filtro comb, diezmado, bandas del plegado, compensación.

1 Introduction

Currently, the design of the most popular analog-to-digital (A/D) converter is based on oversampling and delta-sigma (Δ/Σ) modulation techniques that results in a simpler analog filter, while requiring a more complex digital structure [1]. The key part of this structure is a decimation filter, which is needed to decrease the sampling rate of the oversampled signal. The most popular decimation filter is the comb filter due its computational efficiency and simple implementation requiring only additions/subtractions.

The transfer function of a *K*th-order decimation comb filter can be expressed either in a recursive form or a non-recursive form as

$$H_{COMB}(z)=\left[\frac{1}{M}\left(\frac{1-z^{-M}}{1-z^{-1}}\right)\right]^K=\left(\frac{1}{M}\sum_{n=0}^{M-1}z^{-n}\right)^K, \qquad (1)$$

where M is the decimation factor and K is the number of the comb stages. Some authors call the filter of Eq. 1 CIC (cascaded-integrator-comb) filter, although the CIC filter is referred to a structure proposed by Hogenauer [4].

The magnitude response of the filter of Eq.1 is

$$\left|H_{COMB}(e^{j\omega})\right|=\left|\frac{1}{M}\frac{\sin(\omega M/2)}{\sin(\omega/2)}\right|^K \qquad (2)$$

Various methods have been proposed to model the power consumption of the recursive and non-recursive structures of Eq. 1 ([1], [2], [3],

[12]). Abbas *et al*. extend previous models for the estimation of power and complexity of recursive and non-recursive comb filters and provides a guide-line of the power and implementation complexity by taking into account several factors [1].

The comb filter is usually used at the first decimation stage, whereas a second decimator block with a decimation factor R, which is smaller than that of the comb filter, determines the passband edge frequency ω_p

$$\frac{\omega_p}{\pi} = \frac{1}{RM}, \tag{3}$$

where the worst-case passband distortion occurs [10]. Similarly in [10], the worst-case aliasing is at the frequency

$$\frac{\omega_A}{\pi} = \frac{2}{M} - \frac{1}{RM} = \frac{2R-1}{RM}. \tag{4}$$

In this paper we consider the minimum value for R, i.e.,R = 2.

The comb filter must have a high alias rejection around its zeros (the folding bands) and a low passband droop in the pass band defined by Eq. 3 in order to avoid the distortion of the decimated signal. However, the comb filter has a high passband droop and low folding band attenuations. The first folding band is the most critical, because it has less attenuation than the sub sequential folding bands.

Various methods have been advanced to solve the above two problems. In order to improve the attenuation around the folding bands, the zero-rotation terms have been introduced in [11] and [13]. Fairly simple compensation filters have been proposed in [6, 7] and [9] for improving the comb passband. The sharpening based methods improve the passband and the stopband at the expense of the increased complexity in [5] and [10].

Recently, in [8], a simple two-stage structure with a good aliasing rejection characteristic and a low passband droop for R = 8 have been advanced.

The main goal of this work is to generalize this result so as to provide a low passband droop and high stopband attenuation in the first folding band for a minimum value of R.

The paper is organized as follows. Section 2 describes the two-stage structure and the choice of the design parameters. The design of the compensation filter is given in Section 3. Finally, Section 4 provides a discussion of the results.

2 Two-Stage Structure

We consider here the case where the decimation factor M can be expressed as a product of two integers as $M = M_1 M_2$.

The transfer function of the corresponding comb-based decimation filter is given by

$$H(z) = \left[H_1(z)\right]^{K_1} \left[H_2(z^{M_1})\right]^{K_2}, \tag{5}$$

where

$$\begin{aligned} H_1(z) &= \frac{1}{M_1}\left(\frac{1-z^{-M_1}}{1-z^{-1}}\right), \\ H_2(z^{M_1}) &= \frac{1}{M_2}\left(\frac{1-z^{-M_1 M_2}}{1-z^{-M_1}}\right). \end{aligned} \tag{6}$$

The corresponding magnitude response is given as

$$\begin{aligned} &\left|H(e^{j\omega})\right| = \\ &= \left|\frac{1}{M_1}\frac{\sin(\omega M_1/2)}{\sin(\omega/2)}\right|^{K_1} \left|\frac{1}{M_2}\frac{\sin(\omega M/2)}{\sin(\omega M_1/2)}\right|^{K_2} \end{aligned} \tag{7}$$

2.2.1Choice of Design Parameters

2.2.1.1 Choice of $M_1$and M_2

The choice of M_1 is a matter of compromise between two factors: having less complex polyphase components in the first stage and making the filter in the second stage working as much as possible at a lower rate [8]. To this end, we propose that the factors M_1 and M_2 be close in values as much as possible to each other with $M_1 \leq M_2$.

2.2.1.2 Choice of K_1 and K_2

Next we discuss the benefit of choosing different values for K_1 and K_2 and how to relate them. From Eq. 6 it can be seen that the filter $H_2(z^{M_1})$ has the same zeros as the corresponding comb filter with a decimation factor $M = M_1 M_2$. It follows that for $K = K_2$, both filters will have approximately the same first folding band characteristics. However, a decrease in the attenuation in the lateral folding bands may appear, because K_1 is not equal to K_2.

As an example, Fig. 1 shows the gain responses of the filter of Eq. 5 with $M_1 = 4$, $M_2 = 7$, $K_1 = 4$ and $K_2 = 5$ along with that of comb filter of Eq. 1 with $M = 28$, $K = 4$ and $K = 5$. Note that the filter (5) has the first folding band width almost the same as the comb filter (1) with $K = 5$. However, the filter (5) has 4 cascaded less complex comb filters working at the high input rate, while the comb filter (1) with $K = 5$ has 5 cascaded more complex comb filters working at the high input rate.

Consequently, we can increase K_2 to obtain more attenuation in the first folding band and decrease K_1 to arrive at a less complex overall filter, considering that all subsequent folding bands should have at least the minimum attenuation as the first folding band.

As another example, Fig. 2 illustrates the case where $K_1 = 4$ and $K_2 = 10$. Note that the first folding band has the same attenuation as that of the comb filter with $K = 10$. However, the seventh folding band has less attenuation than the first one.

Therefore the appropriate relation between K_1 and K_2 will prevent that the lateral folding bands have less attenuation than the first folding band.

Using MATLAB simulations we arrive at:

$$K_1 \geq \lfloor K_2 / 2 \rfloor + 1. \tag{8}$$

For example, for $K_2 = 10$ from Eq. (8) it follows that the minimum value for K_1 is equal to 6. Fig.3 shows the gain responses of the proposed filter with $M_1 = M_2 = 4$, K_1=6, K_2=10, and comb filter with M=16, K=6 and 10.

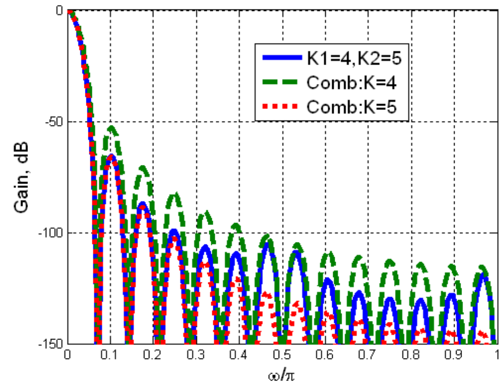


Fig. 1. $M_1 = 4$, $M_2 = 7$, K_1=4, K_2=5

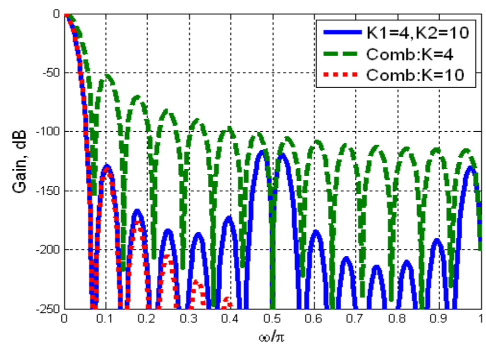


Fig. 2. $M_1 = 4$, $M_2 = 7$, K_1=4, K_2=10

Note the following:

- The proposed filter (5) with K=6, and K_2=10 has almost the same first folding band attenuation, as the original comb filter (1) with K=10.
- However, the comb filter with K=10 has the cascade of 10 filter at high input rate, as a difference to the proposed filter, which has only 6.
- Both, the proposed and comb filters with K = 6, have the cascade of 6 filters at high input rate. However, the attenuation in the first folding band of the proposed filter is much better.

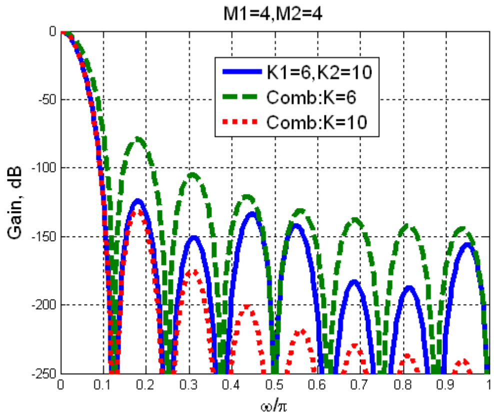


Fig. 3. $M_1 = 4$, $M_2 = 4$, K_1=6, K_2=10

2.2.1.3. Worst-case aliasing attenuation

Next we investigate the worst case aliasing attenuation for the comb filters. Consider the worst-case frequency ω_A, given in (4), for *R*=2,

$$\omega_A = \frac{3\pi}{2M}. \tag{9}$$

Placing (9) into (2), we obtain the worst case aliasing attenuation for the comb filter:

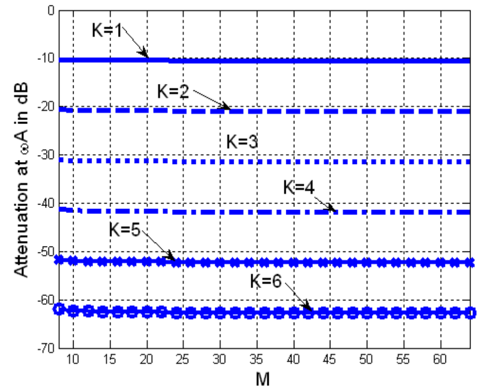
$$\left|H_{COMB}(e^{j\omega_A})\right| = \left|\frac{sin(\omega_A M/2)}{M\, sin(\omega_A/2)}\right|^K = \left|\frac{sin(3\pi/4)}{M\, sin(3\pi/4M)}\right|^K. \tag{10}$$

Fig. 4.a presents the worst-case aliasing attenuations (10) in dB, for *M*=8,…,64, and for different values of *K*, *K* = 1,…,6. The zoom for *K*=6 is shown in Fig.4.b.

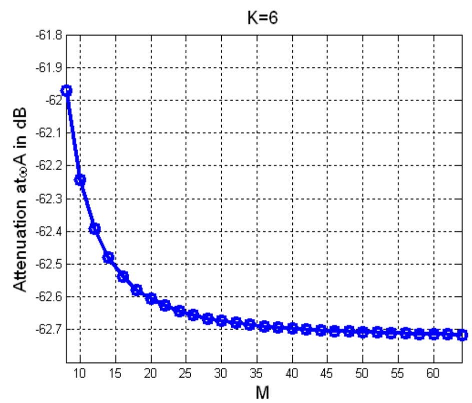
Note that the worst case aliasing attenuation practically does not depend on *M*. The same observation is confirmed mathematically, as explained in the following.

Considering $3\pi/4M$ being small, we can rewrite Eq. 10 as

$$\left|H_{COMB}(e^{j\omega_A})\right| \approx \left|\frac{sin(3\pi/4)}{M(3\pi/4M)}\right|^K = \left|\frac{sin(3\pi/4)}{3\pi/4}\right|^K. \tag{11}$$



a. Worst-case aliasing for different *M* and *K*



b. Zoom for *K*=6

Fig. 4. Worst-case aliasing

For higher values of *M*, the expression (11) becomes more correct, as shown also in Fig.4.b.

As given in Sub-section 2.2.1.2, the parameter K_2 of the proposed filter should be equal to the parameter *K* of the comb filter, in order to have approximately equal worst-case attenuation in the first folding band.

Table 1 shows tentative value of the minimum worst–case aliasing attenuations from Fig.4 and the corresponding values of K_2. The minimum values of K_1, obtained from (8) are also shown.

The method is illustrated by the following example.

Example 1: Let *M* = 15. We choose M_1 = 3 and M_2 = 5, and we want the minimum worst case aliasing attenuation of -60 dB. From Table 1, we choose K_2 = 6, and K_1 = 4. The corresponding

Table 1. Choice of parameters K_1 and K_2 for the desired minimum worst-case aliasing attenuation

Minimum attenuation in ω_A in dB	K_2	Minimum value for K_1
-10	1	2
-20	2	2
-30	3	2
-40	4	3
-50	5	3
-60	6	4

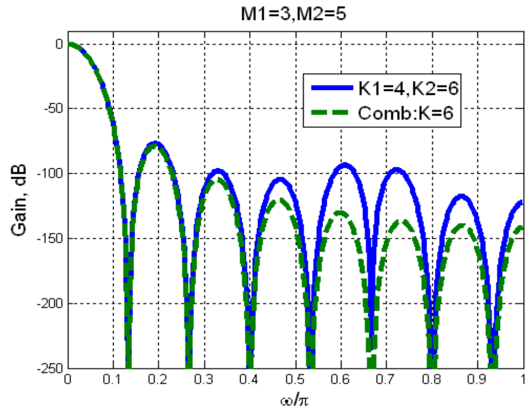


Fig. 5. Illustration of Example 1

gain responses are given in Fig. 5, along with that of the comb filter with M = 15 and K = 6.

Next issue is the improvement of the passband characteristics.

3 Compensation Filter

For small values of ω, sinusoidal functions in the first term of (7) can be approximated as $sin(\alpha) \approx \alpha$ resulting in:

$$G(z^M) = -2^{-4}\left[z^{-M} - (2^4+2)z^{-2M} + z^{-3M}\right] \quad (15)$$

$$\left|H(e^{j\omega})\right| \approx \left|\frac{\omega M_1/2}{M_1\omega/2}\right|^{K_1}\left|\frac{sin(\omega M/2)}{M_2 sin(\omega M_1/2)}\right|^{K_2} = $$
$$= \left|\frac{sin(\omega M/2)}{M_2 sin(\omega M_1/2)}\right|^{K_2} = \left|H_2(e^{j\omega M_1}\right|^{K_2}. \quad (12)$$

This equation shows that the passband of the proposed filter of Eq. 5 is determined by the filter H_2. Additionally, for small values of ω the magnitude characteristic of H_2 can be approximated as:

$$\left|H_2(e^{j\omega M_1})\right|^{K_2} \approx$$
$$\approx \left|\frac{sin(\omega M/2)}{M_2 M_1\omega/2}\right|^{K_2} = \left|\frac{sin(\omega M/2)}{M\omega/2}\right|^{K_2}. \quad (13)$$

Similarly, the magnitude characteristic of the comb filter (1) can be approximated for small values of ω as:

From Equations 13 and 14, it is evident that the passband droop of the proposed filter will be approximately the same as that of the comb filter of Eq. 1, if $K = K_2$. The same is confirmed in Fig. 6, where the passband droops for the filters of Eq. 1 and H_2 are presented for different values of $K_2 = K$.

As a result, we can adopt the comb compensator for the proposed filter (5), just replacing K with K_2. To this end we make use the compensator filter (7):

The filter is multiplierless, and its coefficients are independent of M and K. However, the filter must be cascaded K_3 times, where K_3 depends on K.

Now, the filter can be moved to the lower sampling rate to become the filter $G(z)$. Using analysis from this section and results from [7] we have

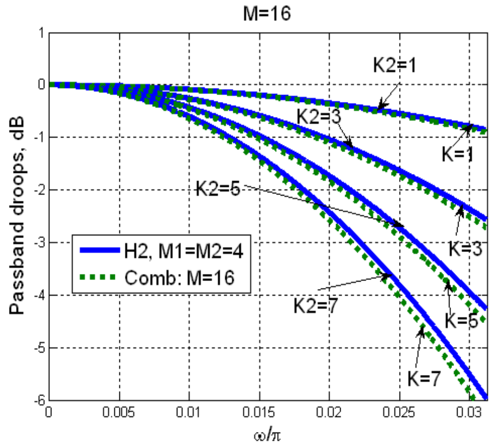


Fig. 6. Passband droops of filters comb and H_2

$$K_3 = \begin{cases} K_2 & \text{for} \quad K_2 \le 3, \\ K_2 - 1 & \text{for} \quad K_2 > 3. \end{cases} \tag{16}$$

The transfer function of the compensated proposed filter is given as

$$H_p(z) = [H_1(z)]^{K_1} [H_2(z^{M_1})]^{K_2} [G(z^M)]^{K_3}. \tag{17}$$

The corresponding structure is shown in Fig. 7.

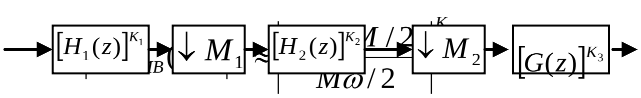


Fig. 7. Proposed structure

Note that the structure includes two decimation stages consisting of the simpler comb decimation filters. The filters can be implemented in either recursive or non-recursive form as explained in [1].

4 Discussion and Results

In this section, we compare the performance of the proposed structure with those of some other methods of similar complexity.

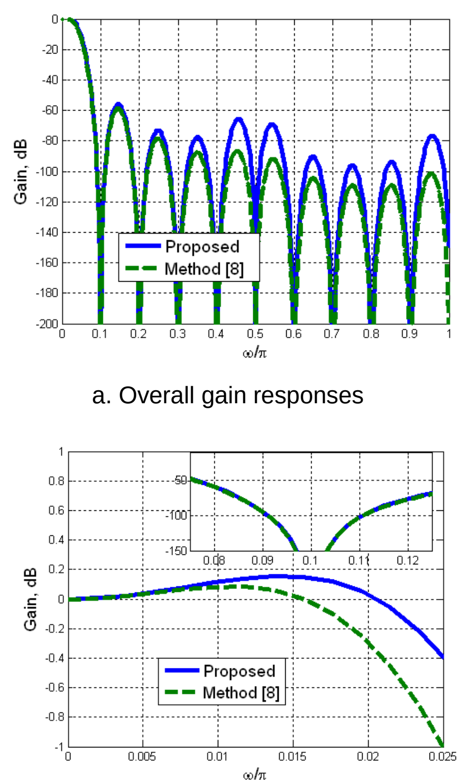
Comparison to the method in [8]

Consider the design of comb decimation filter for M=20 with the minimum alias attenuation of -50dB. From Table 1, we have the following parameters for the proposed method: $K_1 = 3$, $K_2 = 5$, and from (16), K_3=4. The parameters in method [8] are: $K_1 = 4$, $K_2 = 5$, $b = 0$.In both methods M_1=4 and M_2=5.

The gain responses along with the passband and the first folding band zooms are shown in Figures 8.a and b, respectively.

Note that the proposed method and the method in [8] provide the same attenuation in the first folding band. However, the method in [8] has 4 combs at high input rate, compared with the proposed filter which has only 3 combs.

In the proposed method, the side lobes are increased. However, the minimum desired attenuations in all folding bands are satisfied. Additionally, the pass band droop is less than in [8].



a. Overall gain responses

b. Passband and the first folding band zooms

Fig. 8. Comparison with the method in [8]

Comparison to the Sharpening method in [10]

The sharpening method in [10] uses the sharpening polynomial $3H^2$-$2H^3$, where H is the comb filter (1), to simultaneously improve both, the passband and the stopband of the comb filter. In this example we consider M=25 and K =2. In the proposed method, M_1=M_2=5, and K_1=4, K_2=6 and K_3=5.

The overall gainresponses are shown in Fig. 9a, while the pass band and the first folding band zooms are given in Fig.9b.

Note that the proposed method exhibits much better magnitude characteristic in both, the pass band as well as in the stop band. Additionally, the sharpening structure is more complex than the proposed one.

Comparison to the zero-rotation method in [13]

In order to improve the folding band attenuations in [13], the zero-rotation filterhas been introduced as

$$H_r(z) = \frac{1}{M^2} \frac{1 - 2\cos(\alpha M) z^{-M} + z^{-2M}}{1 - 2\cos(\alpha) z^{-1} + z^{-2}} \tag{18}$$

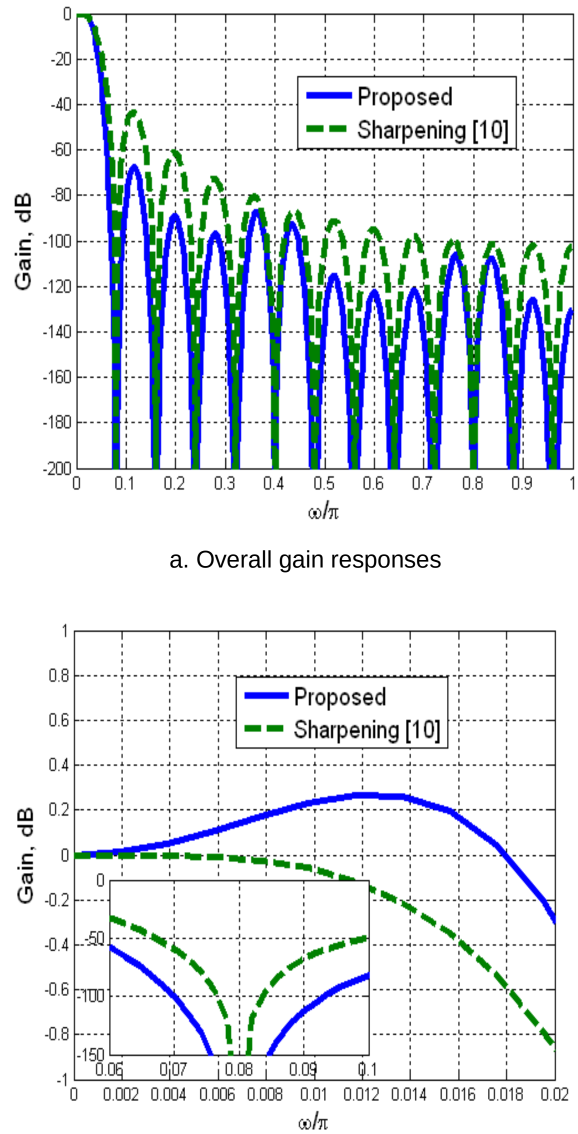
where α is the zero rotation.

The filter (18) is cascaded with the comb filter, and this cascade is referred to as Rotated Sinc (RS) filter:

$$H_{RS}(z) = H_{COMB}(z) H_r(z) \tag{19}$$

Consider the design of RS filter for M=16, and K=3 and α=0.0184. In the proposed method, M_1=M_2=4, K_1=5, and K_2=6, K_3=5.

The overall gain responses for the proposed and RS filter are shown in Fig.10a; the passband and the first folding band zooms are shown in Fig. 10b. Observe that the first folding band aliasing rejections are similar, as well as the attenuations in the sub sequential folding bands. However, the RS method requires two multipliers, one working at high input rate.



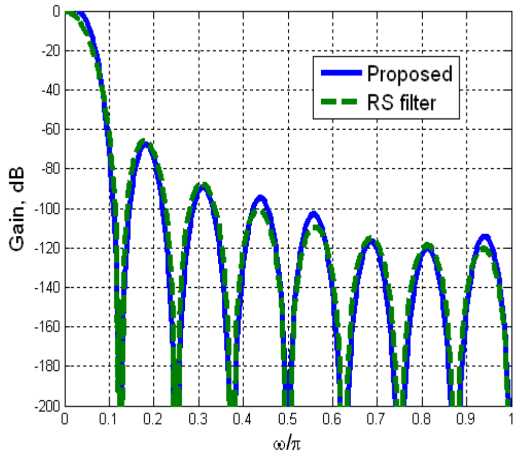
a. Overall gain responses

b. Pass band and the first folding band zooms

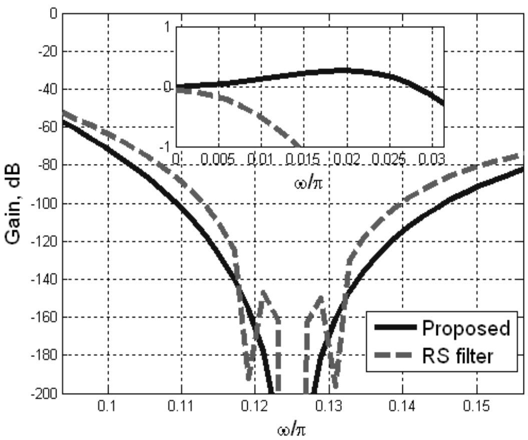
Fig. 9. Comparison to the Sharpening method in [10]

5 Conclusion

We have presented a simple method to improve alias rejection of the first comb folding band. In contrast with the methods, that make use of the zero-rotation term, the proposed method is



a. Overall gain responses



b. Pass band and the first folding band zooms

Fig. 10. Comparison to the RS filter

lesscomplex because it does not require multipliers, while having similar folding band attenuations. Also, the method has a less complexity comparing to the Sharpening and other two-stage methods.

The method is based only on comb filters. Consequently, there is flexibility in the use of a recursive or non-recursive comb structure to achieve different goals regarding to the power consumption and complexity as explained in [1].

The compensation filter $G(z)$ remains the same for all values of M and K_2. However, the number of the cascaded filters K_3 is related to the parameter K_2. If we wish to avoid multiple cascades of the compensator, we may use the compensator from [9]. However, its parameters are different for different values of K_2.

References

1. **Abbas, M., Gustafsson, O., &Wanhammar, L. (2010).**Power estimation of recursive and non-recursive CIC filters implemented in deep-submicron technology. *2010 International Conference on GreenCircuists and Systems (ICGCS 2010)*, Shanghai, China, 221–225.
2. **Aboushadyy, H., Dumonteix, Y., Louerat, M.M., &Mehrez, H. (2001).**Efficient polyphase decomposition of Comb decimation filters in ΣΔ analog-to-digital converters. *IEEE Transactions on Circuits and Systems II*: *Analog and Digital Signal processing,* 48(10), 898–903.
3. **Gao, Y., Jia, L., Isoaho, J., &Tenhunen, H. (2000).**A comparison design of comb decimators for sigma-delta analog-to-digital converters. *Analog Integrated Circuits and Signal Processing*, 22(1), 51–60.
4. **Hogenauer, E.B. (1981).**An economical class of digital filters for decimation and interpolation. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 29(2), 155–162.
5. **Jovanovic-Dolecek, G., & Mitra, S.K. (2005).** A New Two-stage Sharpened Comb Decimator. *IEEE Transactions on Circuits and Systems I: Regular Papers,* 52(7), 1414–1420.
6. **Dolecek, G.J., & Mitra, S.K. (2008).** A Simple Method for the Compensation of CIC Decimation Filter. *Electronics Letters*, 44(19), 1162–1163.
7. **Dolecek, G.J. (2009).** Simple wideband CIC compensator. *Electronics Letters,* 45(24),1270–1272.
8. **Dolecek, G.J., & Mitra, S.K. (2010).** Two-Stage CIC Based Decimator with Improved Characteristics. *IET Signal Processing,* 4(1), 22–29.
9. **Dolecek, G.J., & Dolecek, L. (2010).**Novel Multiplierless Wide-Band CIC Compensator. *InternationalSymposium on Circuits & Systems, (ISCAS 2010),* Paris, France*,* 2119–2122*.*
10. **Kwentus A. & Willson, Jr. A., (1997).**Application of filter sharpening to cascaded integrator-comb decimation filters. *IEEE Transactions on Signal Processing*, 45(2), 457–467.

11. **Laddomada, M. (2007).** Generalized comb decimation filters for ΣΔ A/D converters: Analysis and design. *IEEE Transactions Circuits and Systems I: Regular Papers*, 54(5), 994–1005.
12. **Mortazavi, S.M., Fakhraie, S.M., &Shoaei, O. (2005).**A comparative study and design of decimation filter for high-precision audio data converters. *The 17th International Conference on Microelectronics (ICM 2005)*, Islamabad, Pakistan, 139–143.
13. **Presti, L. L. (2000).**Efficient modified-sinc filters for sigma-delta A/D converters. *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal processing*, 47(11), 1204–1213.

Gordana Jovanovic Dolecek received a B.Sc. degree from the Department of Electrical Engineering, University of Sarajevo, an M.Sc. degree from University of Belgrade, and a Ph.D. degree from the Faculty of Electrical Engineering, University of Sarajevo. She was a professor at the Faculty of Electrical Engineering, University of Sarajevo until 1993, and in 1993-1995 she was with the Institute Mihailo Pupin, Belgrade. In 1995 she joined the Institute INAOE, Department for Electronics, Puebla, Mexico, where she works as a full professor. During 2001-2002 and 2006 she was with Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher. She was with San Diego State University as visiting researcher on a sabbatical leave in 2008-2009. She is the author/co-author of four books, editor of one book, and author/coauthor of 52 journal papers more than 250 conference papers and 19 book chapters. Her research interests include digital signal processing and digital communications. She is a Senior member of IEEE, a member of Mexican Academy of Science, and a member of National System of Researchers (SNI), Mexico.

Sanjit K. Mitra received a B.Sc. (Hons.) degree in Physics from the Utkal University in 1953, the M.Sc. (Tech.) degree in Radio Physics and Electronics from the Calcutta University in 1956, and the M.S. and Ph.D. degrees in Electrical Engineering from the University of California, Berkeley, in 1960 and 1962, respectively. He has been a Professor of Electrical and Computer Engineering at the University of California, Santa Barbara since 1977, where he served as Chairman of the Department from July 1979 to June 1982. He has served IEEE in various capacities including service as the President of the IEEE Circuits and Systems Society in 1986 and as a Member-at-Large of the Board of Governors of the IEEE Signal Processing Society in 1996-1999. He has published over 600 papers in signal and image processing, twelve books, and holds five patents. He is a member of the U.S. National Academy of Engineering, an Academician of the Academy of Finland, a member of the Norwegian Academy of Technological Sciences, a foreign member of the Croatian Academy of Sciences and Arts, and a foreign member of the Academy of Engineering of Mexico. He has been awardedhonorary doctorate degrees from the Tampere University of Technology, Finland, and the Politehnica University of Bucharest, Romania. Dr. Mitra is a Fellow of the IEEE, AAAS, and SPIE, and a member of EURASIP.

Article received on 13/06/2011; accepted on 29/08/2011.