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Modelling and Verification Analysis of Cooperative and Non-Cooperative Games via a Modal Logic Approach

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Abstract. In game theory, a cooperative game (or coalitional game) is a game with competition between groups of players (coalitions) due to the possibility of external enforcement of cooperative behavior (e.g. through contract law). Those are opposed to non-cooperative games in which there is either no possibility to forge alliances or all agreements need to be self-enforcing (e.g. through credible threats). Cooperative games are often analyzed through the framework of cooperative game theory, which focuses on predicting which coalitions will form, the joint actions that groups take and the resulting collective payoffs. It is opposed to the traditional non-cooperative game theory which focuses on predicting individual players' actions and payoffs and analyzing Nash equilibriums. In this work, the cooperative and non-cooperative game problem is modeled by means of a modal logic formula. Then, using the concept of logic implication, and transforming this logical implication relation into a set of clauses, a modal resolution qualitative method for verification (satisfiability) as well as performance issues, for some queries is applied.

Keywords. Cooperative game, non-cooperative game, modal logic, model, verification, unsatisfiability, modal resolution method.

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1 Introduction

In game theory, a cooperative game (or coalitional game) is a game with competition between groups of players (coalitions) due to the possibility of external enforcement of cooperative behavior (e.g. through contract law). Those are opposed to non-cooperative games in which there is either no possibility to forge alliances or all agreements need to be self-enforcing (e.g. through credible threats).

In this study, cooperative games are often analyzed through the framework of cooperative game theory, which focuses on predicting which coalitions will form, the joint actions that groups take and the resulting collective payoff, while non-cooperative games have been studied using traditional non-cooperative game theory which focuses on predicting individual players' actions and payoffs and analyzing Nash equilibriums.

This paper proposes a well defined syntax modeling and verification analysis methodology which consists in representing the biological competition system as a modal logic formula.

This approach allows to represent both cases, the cooperative and non cooperative ones, in one formula and not as two separate formulas and, it also models other behavioral possibilities not always easy to represent using other techniques. The modal logic approach introduces two new operators that enable abstract relations like necessarily true and possibly true to be expressed directly, called alethic modalities, what is not possible using first order logic.

For example, the statement: 7 is a prime number, is necessarily true always and everywhere, in contrast, the statement the head of state of this country is a king is possibly true, because its truth changes from place to place and from time to time. Other modalities that have been formalized in modal logic include temporal modalities, or modalities of time, deontic modalities, epistemic modalities, and doxastic modalities.

The main idea consists in modeling the biological competition system by means of a modal logic formula. Then, using the concept of logic implication, and transforming this logical implication relation into a set of clauses, a modal resolution qualitative method for verification (satisfiability) as well as performance issues, for some queries is applied. The paper is organized as follows. In section 2, a modal logic background summary is given. In section 3, the modal resolution principle for unsatisfiability, is recalled. In section 4, the biological competition problem is addressed. The cooperative and non cooperative cases are considered. Finally, the paper ends with some conclusions.

2 Modal Logic Background

This section presents a summary of modal logic theory. The reader interested in more details is encouraged to see [1, 2].

Definition 1 *A modal language $\mathcal{L}$ is an infinite collection of distinct symbols, no one of which is properly contained in another, separated into the following categories: parentheses, connectives, possibility modality, necessity modality, proposition variables $\Phi_0 = \{p_1, p_2, \cdots\}$ (called atoms), contradiction(falsity), true(tautology).*

Definition 2 *Well-formed formulas, or formulas for short, in modal logic are defined recursively as follows:(i). An atom is a formula, $\perp$ (false is a formula), T (true is a formula) (ii). If F and G are formulas then, $\sim (F)$, $(F \vee G)$, $(F \wedge G)$, $(F \leftrightarrow G)$, $\Box F$, $\diamond F$, are formulas. $\diamond A \equiv \sim \Box \sim A$. Formulas are generated only by a finite number of applications of (i) and (ii), therefore the set of welled formed formulas is enumerable infinite.*

Remark 3 *It is important to underline the unique readability of the formulas which is secured by the assumption that the operators are one to one.*

Definition 4 *A Kripke frame (frame) $\mathcal{F}$ is a pair $(W, \mathcal{R})$ in which W is a set of worlds (time, states, etc), and $\mathcal{R} \subseteq \mathcal{W} \times \mathcal{W}$ is a binary relation over W.*

Definition 5 *A Kripke model (model) $\mathcal{M}$ over frame $\mathcal{F}$ is a triple $(\mathcal{F}, \pi) = (\mathcal{W}, \mathcal{R}, \pi)$ where $\pi : \Phi_0 \to 2^W$ the set of worlds where each element of Φ_0 is true is an assignment or interpretation.*

Definition 6 *Given any model $\mathcal{M}$, a world $w \varepsilon W$, the notion of true at w is defined as follows:*

— $\mathcal{M}, w \models p_n \Leftrightarrow w \varepsilon \pi(p_n), n = 1, 2, \cdots,$

— $\mathcal{M}, w \models \sim F \Leftrightarrow w \not\models F$,

— $\mathcal{M}, w \models F \wedge G \Leftrightarrow w \models F$ *and* $w \models G$,

— $\mathcal{M}, w \models F \vee G \Leftrightarrow w \models F$ *or* $w \models G$,

— $\mathcal{M}, w \models F \to G \Leftrightarrow$ *if* $w \models F$ *then* $w \models G$,

— $\mathcal{M}, w \models F \equiv G \Leftrightarrow w \models F$ *iff* $w \models G$,

— $\mathcal{M}, w \models \diamond F \Leftrightarrow$ *there exists* $u \varepsilon W$ *such that* $(w, u) \varepsilon \mathcal{R}, \mathcal{M}, u \models F$,

— $\mathcal{M}, w \models \Box F \Leftrightarrow$ *for all* $u \varepsilon W$ *such that* $(w, u) \varepsilon \mathcal{R}, \mathcal{M}, u \models F$.

Definition 7 *A formula F is consistent (satisfiable, true at w) in a model $\mathcal{M}$ in a world $w \varepsilon W$ iff $\mathcal{M}, w \models F$, then we say that $\mathcal{M}$ is a. model for F. If this happens for all worlds $w \varepsilon W$ then we say it is true.*

Definition 8 *A formula F is inconsistent (unsatisfiable) in a model $\mathcal{M}$ iff $\mathcal{M}, w \not\models F$ for every world $w \varepsilon W$, then we say that $\mathcal{M}$ is a countermodel for F.*

Definition 9 *A formula F is valid in a class of models $\mathcal{CM}$ if and only if it is true for all models in the class. This will be denoted by $\models_{\mathcal{CM}} F$.*

Definition 10 *A formula F is valid iff it is valid for every class of models $\mathcal{CM}$. This will be denoted by $\models F$.*

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Definition 11 *A formula G is a logical implication of formulas $F_1, F_2, \ldots, F_n$ if and only if for every model $\mathcal{M}$, that makes $F_1, F_2, \ldots, F_n$ true, G is also true in $\mathcal{M}$.*

The following characterization of logical implication plays a very important role as will be shown in the rest of the paper.

Theorem 12 *Given formulas $F_1, F_2, \ldots, F_n, G$, G is a logical implication of $F_1, F_2, \ldots, F_n$ if and only if the formula $((F_1 \wedge F_2 \wedge \ldots, \wedge F_n) \rightarrow G)$ is valid in a class of models if and only if the formula $(F_1 \wedge F_2 \wedge \ldots \wedge F_n \wedge \sim (G))$ is unsatisfiable.*

Proof. Setting the class of models equal to all the models that make $F_1 \wedge F_2 \wedge \ldots, \wedge F_n$ true. The first iff follows directly by the definition of validity in a class of models, and logical implication.

For the second one, since $F_1 \wedge F_2 \wedge \ldots, \wedge F_n \rightarrow G$ is valid in a class of models, every model that makes $F_1 \wedge F_2 \wedge \ldots, \wedge F_n$ true does not satisfy $\sim (G)$, therefore $(F_1 \wedge F_2 \wedge \ldots \wedge F_n \wedge \sim (G))$ can not be satisfied. Reversing this last argument we obtain the last implication. ∎

Next, given a class of models $\mathcal{CM}$, we define the syntactic mechanisms capable of generating the formulas valid on $\mathcal{CM}$.

Axioms:

(1). All instances of propositional logic tautologies,
(2). $\Box(F \rightarrow G) \rightarrow \Box F \rightarrow \Box G$.

Rules of inference:

(1). Modus ponens:

$$\frac{F, F \rightarrow G}{G},$$

(2). Necessitation

$$\frac{F}{\Box G}.$$

We write $\vdash F$ if F can be deduced from the axioms and the inference rules.

Theorem 13 *(Completeness [1]) A formula F is valid iff it is provable i.e., $\models F \Leftrightarrow \vdash F$'.*

Definition 14 *A formula F in modal logic is said to be in disjunctive normal form normal (DNF) if and only if is a disjunction (perhaps with zero disjunct) of the form $F == L_1 \vee L_2 \vee \cdots L_n \vee \Box D_1 \vee \Box D_2 \vee \cdots \Box D_m \vee \diamond H_1 \vee \diamond H_2 \vee \cdots \diamond H_j$, where each L_i is an atom or its negation, each D_i is a DNF, and each H_i is a CNF (next defined). A formula G is said to be in conjunctive normal form (CNF) if it is a conjunction of F_i DNF i.e., $G = F_1 \wedge F_2 \wedge \cdots \wedge F_n$ which will be denoted by the set $G = \{F_1, F_2, \ldots, F_n\}$*

Definition 15 *A formula in DNF is called a clause. A clause with only one element is called a unit clause. A clause with zero disjunct is empty and it will be denoted by the $\perp$ symbol. Since the empty clause has no literal that can be satisfied by a model, the empty clause is always false.*

Definition 16 *The modal degree of a formula F denoted by $d(F)$ is recursively defined as follows:*

— *if F is a literal then its degree is zero,*

— *$d(F \triangle G) = max(d(F), d(G))$, where $\triangle$ is $\wedge$ or $\vee$,*

— *$d(\sim F)) = d(F)$,*

— *$d(\nabla F) = d(F) + 1$, where ∇ stands for $\Box$ or $\diamond$.*

Given a formula F, the following inductive procedure transforms F into a CNF in such a way that the original formula is equal to its CNF form therefore satisfying validity:

1. Using axioms 1 and 2, the definition $\sim \Box F \equiv \diamond \sim F$ and the inference rules, eliminate all propositional other than $\wedge, \vee, \sim$ and move negations inside so that they are immediately before propositional variables,

2. If $d(F) = 0$ then apply the propositional procedure [3],

3. If $F = \Box F_1$ with F_1 in CNF, apply the theorem $\Box(F \wedge G) \equiv \Box F \wedge \Box G$ to distribute the $\Box$ operator (this is proved with the aid of axiom 2).
4. If $F = \diamond F_1$ with F_1 in CNF, then do not do anything.
5. Otherwise, we have a combination of different formulas which can be handled using the preceding rules.

Therefore, we have proved the following result.

Theorem 17 *Let S be a set of clauses that represents a formula F in its CNF. Then F is unsatisfiable if and only if S is unsatisfiable.*

3 The Modal Logic Resolution Principle

We shall next present the resolution principle inspired by the propositional logic resolution principle introduced by Robinson (see [3], the references quoted therein, and [4]). It can be applied directly to any set S of clauses to test the unsatisfiability of S.

Resolution is a decidable, sound and complete proof system i.e., a formula in clausal form is unsatisfiable if and only if there exists an algorithm reporting that it is unsatisfiable. Therefore it provides a consistent methodology free of contradictions. It is composed of rules for computing resolvents, simplification rules and rules of inference. The first ones compute resolvents, simplified by the simplification rules, and then inferred by the rules of inference.

Definition 18 *[4] Let $\Sigma(A, B) \to C$, and $\Gamma(A) \to C$ be two relations on clauses defined by the following formal system:*

Axioms:

(1). $\Sigma(p, \sim p) \to \perp$,
(2). $\Sigma(\perp, A) \to \perp$.

Σ rules:

$$\vee - \text{rule} : \frac{\Sigma(A, B) \to C}{\Sigma(A \vee D_1, B \vee D_2) \to C \vee D_1 \vee D_2},$$

$$\Box \diamond - \text{rule} : \frac{\Sigma(A, B) \to C}{\Sigma(\Box A, \diamond(B, E)) \to \diamond(B, C, E)},$$

$$\Box\Box - \text{rule} : \frac{\Sigma(A, B) \to C}{\Sigma(\Box A, \Box B) \to \Box C}.$$

Γ rules:

$$\diamond - \text{rule} \quad 1 : \frac{\Sigma(A, B) \to C}{\Gamma(\diamond(A, B, F)) \to \diamond(A, B, C, F)},$$

$$\diamond - \text{rule} \quad 2 : \frac{\Gamma(A) \to B}{\Gamma(\diamond(A, F)) \to \diamond(B, A, F)},$$

$$\vee - \text{rule} : \frac{\Gamma(A) \to B}{\Gamma(A \vee C) \to B \vee C},$$

$$\Box - \text{rule} : \frac{\Gamma(A) \to B}{\Gamma(\Box A) \to \Box B},$$

where A, B, C, D, D_1, D_2, denote general clauses, E, F denote sets (conjunctions) of clauses, and $(A < E)$ denotes the result of appending the clauses A to the set E.

Simplification rules:

The relation 'A can be simplified in B' denoted $A \simeq B$ is the least congruence relation containing: (S1) $\diamond \perp \simeq \perp$, (S2) $\perp \vee D \simeq D$, (S3) $\perp, E \simeq \perp$, (S4) $A \vee A \vee D \simeq A \vee D$. The simplified formula obtained is called the normal form of the original formula and is the one to be considered when computing resolvents.

Inference rules:

(R1).
$$\frac{C}{D} \quad if \quad \Gamma(C) \to D,$$

(R2).
$$\frac{C_1 \quad C_2}{D} \quad \text{if} \quad \Sigma(C_1, C_2) \to D.$$

where C, C_1, C_2, D are general clauses.

A deduction of a clause D from a set S of clauses can be seen as a tree whose root is D, whose leaves are clauses of S, and every internal node C has sons A and B (respectively A) iff the rule R2 (respectively RI) can be applied with premises A and B (respectively A) and conclusion C. The size of a deduction is the number of nodes of this tree.

We say that D is a-consequence of S iff there is a deduction of D from S denoted by $S \vdash D$. These definitions and notations are extended to sets of consequences: if S' is a set of clauses, $S \vdash S^{'}$ iff $S \vdash D$ for every $D \varepsilon S^{'}$. A deduction of $\perp$ from S is a refutation of S.

Theorem 19 *[4] The resolution proof system is decidable.*

The main two results of this subsection: the completeness theorem for the resolution proof system, and that proofs in the resolution proof system are actually proofs in our modal logic axiomatic system are next presented.

Theorem 20 *[4] A set S of clauses is unsatisfiable if and only if there is a deduction of the empty clause $\perp$ from S.*

Theorem 21 *[4] If there exists a deduction D from S in the resolution proof system then there is a deduction D from S in our modal logic axiomatic system.*

4 The Cooperative and Non-Cooperative Game Problem

The biological competition system behavior is described as follows:

1. Propositional variables: S: resources are safe, D: the resources are in danger, B: the resources are being eaten, I_1, I_2: the organisms are inactive, L_1, L_2: the organisms are in search for a resource, CL_1, CL_2: the organisms continue searching for a resource, A_1, A_2: the organisms attack the resource, F_1, F_2: the organisms have finished eating the resource, P_1, P_2: the organisms die, S_1: organism one is stronger than organism two, S_2: organism two is stronger than organism one ; E_1: organism one eliminates organism two, E_2: organism two eliminates organism one;
2. Rules of Inference: (a) if S and L_1 or L_2 then CL_1 or CL_2, (b) if S and CL_1 or CL_2 then $\diamond P_1$ or $\diamond P_2$, (c) if S and CL_1 or CL_2 and $\diamond P_1$ or $\diamond P_2$ then P_1 or P_2 , (d) if D and ((L_1 or CL_1) and not(L_2 or CL_2)) then A_1 and not(A_2), (e) if D and (not(L_1 or CL_1) and (L_2 or CL_2)) then not(A_1) and A_2, (f) if A_1 and not(A_2) then B_1 and not(B_2),(g) if not(A_1) and A_2 then not(B_1) and B_2, (h) if B_1 and not(B_2) then F_1 and not(F_2), (i) if not(B_1) and B_2 then not(F_1) and F_2, (j) if F_1 and not(F_2) then I_1 and not(I_2), (k) if not(F_1) and F_2 then not(I_1) and I_2,(l) if I_1 and not(I_2) then L_1 and not(L_2), (m) if not(I_1) and I_2 then not(L_1) and L_2, (n) if D and ((L_1 or CL_1) and (L_2 or CL_2)) and $\Box S_1$ then A_1 and E_1, (o) if D and ((L_1 or CL_1) and (L_2 or CL_2)) and $\Box S_2$ then A_2 and E_2, (p) if E_1 then not(L_2 or CL_2) and not A_2 and not B_2 and not F_2 and not I_2, (q) if E_2 then not(L_1 or CL_1) and not A_1 and not B_1 and not F_1 and not I_1,(r) if D then $\diamond A_1$ and $\diamond A_2$, (s) if $\diamond A_1$ and $\diamond A_2$ then $\Box A_1$ and $\Box A_2$, (t) if $\Box A_1$ and $\Box S_1$ then A_1 and $E1$, (u) if $\Box A_2$ and $\Box S_2$ then A_2 and E_2, (v) if $\Box A_1$ and $\Box S_2$ then A_2 and E_2, (w) if $\Box A_2$ and $\Box S_1$ then A_1 and E_1.

Remark 22 *It important to underline that the inference rules express the cooperative and non cooperative behavior of the players. In the cooperative case one organism takes control over the resource while the other one stays apart. This cooperative competitive behavior differs from the strictly competitive where there exists just one of the organisms (the winner) who takes completely control of the resource.*

Remark 23 *The main idea consists of: the biological competition system behavior is expressed by a modal logic formula, some query is expressed as an additional formula. The query is assumed to be a logical implication of the biological competition formula (see theorem 12). Then, transforming this logical implication relation into a set of clauses by using the techniques given in section 3, its validity can be checked. It is important to point out that other type of behaviors can be incorporated in to the model by the modeler, making it as close to reality as needed.*

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The formula that models the biological competition system behavior turns out to be Equation 1: $S \wedge L_1 \wedge L_2 \rightarrow CL_1 \wedge CL_2 \wedge [S \wedge CL_1 \wedge CL_2 \rightarrow \diamond P_1 \wedge \diamond P_2] \wedge [D \wedge (L_1 \vee CL_1) \wedge \sim (L_2 \vee CL_2) \rightarrow A_1 \wedge \sim A_2] \wedge [D \wedge \sim (L_1 \vee CL_1) \wedge (L_2 \vee CL_2) \rightarrow \sim A_1 \wedge A_2] \wedge [A_1 \wedge \sim A_2 \rightarrow B_1 \wedge \sim B_2] \wedge [\sim A_1 \wedge A_2 \rightarrow \sim B_1 \wedge B_2] \wedge [(B_1 \wedge \sim B_2 \rightarrow F_1 \wedge \sim F_2)] \wedge [\sim B_1 \wedge B_2 \rightarrow \sim F_1 \wedge F_2] \wedge [F_1 \wedge \sim F_2 \rightarrow I_1 \wedge \sim I_2)] \wedge [\sim F_1 \wedge F_2 \rightarrow \sim I_1 \wedge I_2] \wedge [I_1 \wedge \sim I_2 \rightarrow L_1 \wedge \sim L_2] \wedge [\sim I_1 \wedge I_2 \rightarrow \sim L_1 \wedge L_2] \wedge [D \wedge (L_1 \vee CL_1) \wedge (L_2 \vee CL_2) \wedge \Box S_1 \rightarrow A_1 \wedge E_1] \wedge [D \wedge (L_1 \vee CL_1) \wedge (L_2 \vee CL_2) \wedge \Box S_2 \rightarrow A_2 \wedge E_2]\ [E_1 \rightarrow \sim (L_2 \vee CL_2) \wedge \sim A_2 \wedge \sim B_2 \wedge \sim F_2 \wedge \sim I_2], [E_2 \rightarrow \sim (L_1 \vee CL_1) \wedge \sim A_1 \wedge \sim B_1 \wedge \sim F_1 \wedge \sim I_1], [D \rightarrow \diamond A_1 \wedge \diamond A_2], [\diamond A_1 \wedge \diamond A_2 \rightarrow \Box A_1 \wedge \Box A_2], [\Box A_1 \wedge \Box S_1 \rightarrow A_1 \wedge E1], [\Box A_2 \wedge \Box S_2 \rightarrow A_2 \wedge E_2], [\Box A_1 \wedge \Box S_2 \rightarrow A_2 \wedge E_2], [\Box A_2 \wedge \Box S_1 \rightarrow A_1 \wedge E_1]$.

We are interested in verifying, the following statements:

(S1) Claim: In the cooperative case, we want to verify that in the case when one of the organisms takes control over the resource the other one stays apart i.e., if D and ((L_1 or CL_1) and not(L_2 or CL_2)) then B_1 and not(B_2). Specifically, we want to know if the following formula is a logical implication of equation 1: $D \wedge (L_1 \vee CL_1) \wedge \sim (L_2 \vee CL_2) \rightarrow B_1 \wedge \sim B_2$.

The set of clauses for this case is given by: $S = \{(\sim S \vee \sim L_1 \vee \sim L_2 \vee CL_1), (\sim S \vee \sim L_1 \vee \sim L_2 \vee CL_2), (\sim S \vee \sim CL_1 \vee \sim CL_2 \vee \diamond P_1), (\sim S \vee \sim CL_1 \vee \sim CL_2 \vee \diamond P_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee A_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \sim A_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee A_1), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \sim A_2), (\sim D \vee \sim L_2 \vee L_1 \vee CL_1 \vee \sim A_1), (\sim D \vee L_1 \vee \sim L_2 \vee CL_1 \vee A_2), (\sim D \vee CL_1 \vee L_1 \vee \sim CL_2 \vee \sim A_1), (\sim D \vee CL_1 \vee L_1 \vee \sim CL_2 \vee A_2), (\sim A_1 \vee A_2 \vee B_1), (\sim A_1 \vee A_2 \vee \sim B_2), (\sim A_2 \vee A_1 \vee \sim B_1), (\sim A_2 \vee A_1 \vee B_2), (\sim B_1 \vee B_2 \vee F_1), (\sim B_1 \vee B_2 \vee \sim F_2), (\sim B_2 \vee B_1 \vee \sim F_1), (\sim B_2 \vee B_1 \vee F_2), (\sim F_1 \vee F_2 \vee I_1), (\sim F_1 \vee F_2 \vee \sim I_2), (\sim F_2 \vee F_1 \vee \sim I_1), (\sim F_2 \vee F_1 \vee I_2), (\sim I_1 \vee I_2 \vee L_1), (\sim I_1 \vee I_2 \vee \sim L_2), (\sim I_2 \vee I_1 \vee L_1), (\sim I_2 \vee I_1 \vee L_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee A_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee E_1), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee A_1), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee E_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee A_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee E_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee A_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee E_2), (\sim E_1 \vee \sim L_2), (\sim E_1 \vee \sim CL_2), (\sim E_1 \vee \sim A_2), (\sim E_1 \vee \sim B_2), (\sim E_1 \vee \sim F_2), (\sim E_1 \vee \sim I_2), (\sim E_2 \vee \sim L_1), (\sim E_2 \vee \sim CL_1), (\sim E_2 \vee \sim A_1), (\sim E_2 \vee \sim B_1), (\sim E_2 \vee \sim F_1), (\sim E_2 \vee \sim I_1), (\sim D \vee \diamond A_1), (\sim D \vee \diamond A_2), (\Box \sim A_1 \vee \Box \sim A_2 \vee \Box A_1), (\Box \sim A_1 \vee \Box \sim A_2 \vee \Box A_2), (\diamond \sim A_1 \vee \diamond \sim S_1 \vee A_1), (\diamond \sim A_1 \vee \diamond \sim S_1 \vee E_1), (\diamond \sim A_2 \vee \diamond \sim S_2 \vee A_2), (\diamond \sim A_2 \vee \diamond \sim S_2 \vee E_2), (\diamond \sim A_1 \vee \diamond \sim S_2 \vee A_2), (\diamond \sim A_1 \vee \diamond \sim S_2 \vee E_2), (\diamond \sim A_2 \vee \diamond \sim S_1 \vee A_1), (\diamond \sim A_2 \vee \diamond \sim S_1 \vee E_1), (D), (L_1 \vee CL_1), (\sim L_2), (\sim CL_2), (\sim B_1 \vee B_2)\}$.

Then applying the $\Sigma\ \vee$ rule, a resolution refutation proof for S, is as follows:

(a) $(\sim A_1 \vee A_2 \vee B_1)(\sim B_1 \vee B_2) \rightarrow (\sim A_1 \vee A_2 \vee B_2)$.

(b) $(\sim A_1 \vee A_2 \vee B_2)(\sim A_1 \vee A_2 \vee \sim B_2) \rightarrow (\sim A_1 \vee A_2)$.

(c) $(\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \sim A_2)(D)(\sim L_2)(\sim CL_2) \rightarrow (\sim CL_1 \vee \sim A_2)$.

(d) $(\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \sim A_2)(D)(\sim L_2)(\sim CL_2) \rightarrow (\sim L_1 \vee \sim A_2)$.

(e) $(\sim L_1 \vee \sim A_2)(L_1 \vee CL_1) \rightarrow (CL_1 \vee \sim A_2)$.

(f) $(\sim CL_1 \vee \sim A_2)(CL_1 \vee \sim A_2) \rightarrow (\sim A_2)$.

(g) $(\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee A_1)(D)(\sim L_2)(\sim CL_2) \rightarrow (\sim CL_1 \vee A_1)$.

(h) $(\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee A_1)(D)(\sim L_2)(\sim CL_2) \rightarrow (\sim L_1 \vee A_1)$.

(i) $(\sim L_1 \vee A_1)(L_1 \vee CL_1) \rightarrow (CL_1 \vee A_1)$.

(j) $(\sim CL_1 \vee A_1)(CL_1 \vee A_1) \rightarrow A_1$.

Now, from (b) and (j) we get:

(k) $(\sim A_1 \vee A_2)(A_1) \rightarrow A_2$.

Therefore, from the conclusion of (f) and (k), we get a proof of S i.e., $\perp$.

(S2) Claim: For the non cooperative case, we want to verify that when the resource is in danger and there is a possibility of attack by both organisms, the stronger organism is the one who takes control over the resource, and not being this enough, he decides to eliminate his opponent. Specifically, we want to know if the following formula is a logical implication of equation 1: $D \wedge \diamond A_1 \wedge \diamond A_2 \wedge \Box S_1 \rightarrow A_1 \wedge E_1$.

The set of clauses for this case is given by: $S = \{(\sim S \vee \sim L_1 \vee \sim L_2 \vee CL_1), (\sim S \vee \sim L_1 \vee \sim L_2 \vee CL_2), (\sim S \vee \sim CL_1 \vee \sim CL_2 \vee \diamond P_1), (\sim S \vee \sim CL_1 \vee \sim CL_2 \vee \diamond P_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee A_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \sim A_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee A_1), (\sim D \vee \sim$

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$CL_1 \vee L_2 \vee CL_2 \vee \sim A_2), (\sim D \vee \sim L_2 \vee L_1 \vee CL_1 \vee \sim A_1), (\sim D \vee L_1 \vee \sim L_2 \vee CL_1 \vee A_2), (\sim D \vee CL_1 \vee L_1 \vee \sim CL_2 \vee \sim A_1), (\sim D \vee CL_1 \vee L_1 \vee \sim CL_2 \vee A_2), (\sim A_1 \vee A_2 \vee B_1), (\sim A_1 \vee A_2 \vee \sim B_2), (\sim A_2 \vee A_1 \vee \sim B_1), (\sim A_2 \vee A_1 \vee B_2), (\sim B_1 \vee B_2 \vee F_1), (\sim B_1 \vee B_2 \vee \sim F_2), (\sim B_2 \vee B_1 \vee \sim F_1), (\sim B_2 \vee B_1 \vee F_2), (\sim F_1 \vee F_2 \vee I_1), (\sim F_1 \vee F_2 \vee \sim I_2), (\sim F_2 \vee F_1 \vee \sim I_1), (\sim F_2 \vee F_1 \vee I_2), (\sim I_1 \vee I_2 \vee L_1), (\sim I_1 \vee I_2 \vee \sim L_2), (\sim I_2 \vee I_1 \vee L_1), (\sim I_2 \vee I_1 \vee L_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee A_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee E_1), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee A_1), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_1 \vee E_1), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee A_2), (\sim D \vee \sim L_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee E_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee A_2), (\sim D \vee \sim CL_1 \vee L_2 \vee CL_2 \vee \diamond \sim S_2 \vee E_2), (\sim E_1 \vee \sim L_2), (\sim E_1 \vee \sim CL_2), (\sim E_1 \vee \sim A_2), (\sim E_1 \vee \sim B_2), (\sim E_1 \vee \sim F_2), (\sim E_1 \vee \sim I_2), (\sim E_2 \vee \sim L_1), (\sim E_2 \vee \sim CL_1), (\sim E_2 \vee \sim A_1), (\sim E_2 \vee \sim B_1), (\sim E_2 \vee \sim F_1), (\sim E_2 \vee \sim I_1), (\sim D \vee \diamond A_1), (\sim D \vee \diamond A_2), (\Box \sim A_1 \vee \Box \sim A_2 \vee \Box A_1), (\Box \sim A_1 \vee \Box \sim A_2 \vee \Box A_2), (\diamond \sim A_1 \vee \diamond \sim S_1 \vee A_1), (\diamond \sim A_1 \vee \diamond \sim S_1 \vee E_1), (\diamond \sim A_2 \vee \diamond \sim S_2 \vee A_2), (\diamond \sim A_2 \vee \diamond \sim S_2 \vee E_2), (\diamond \sim A_1 \vee \diamond \sim S_2 \vee A_2), (\diamond \sim A_1 \vee \diamond \sim S_2 \vee E_2), (\diamond \sim A_2 \vee \diamond \sim S_1 \vee A_1), (\diamond \sim A_2 \vee \diamond \sim S_1 \vee E_1), (D), (\diamond A_1), (\diamond A_2), (\Box S_1), (\sim A_1 \vee \sim E_1)\}.$

Then applying the $\Sigma \vee$ rule, the $\Sigma \Box \diamond$ rule and the simplifications rules, a resolution refutation proof for S, is as follows:
(a) $(\Box \sim A_1 \vee \Box \sim A_2 \vee \Box A_1)(\diamond A_1)(\diamond A_2) \rightarrow (\Box A_1)$.
(b) $(\diamond \sim A_1 \vee \diamond \sim S_1 \vee A_1)(\Box S_1) \rightarrow (\diamond \sim A_1 \vee A_1)$.
(c) $(\diamond \sim A_1 \vee A_1)(\Box A_1) \rightarrow (A_1)$.
(d) $(\sim A_1 \vee \sim E_1)(A_1) \rightarrow (\sim E_1)$.
(e) $(\diamond \sim A_1 \vee \diamond \sim S_1 \vee E_1)(\Box S_1) \rightarrow (\diamond \sim A_1 \vee E_1)$.
(f) $(\diamond \sim A_1 \vee E_1)(\Box A_1) \rightarrow (E_1)$.

Therefore, from the conclusion of (d) and (f), we get a proof of S, i.e., $\perp$.

5 Conclusions

The main contribution of the paper consists in the study of cooperative and non- games by means of a formal reasoning deductive methodology based on modal logic theory. The cooperative and non cooperative cases were addressed. Verification (validity) as well as performance issues, for some queries were addressed.

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