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Velocity Estimation by Using a State Reconstructor for Stabilizing a Two Degree of Freedom Mechanical Manipulator

Estimación de Velocidad Usando un Reconstructor de Estado para Estabilizar un Manipulador Mecánico de Dos Grados de Libertad

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Abstract

In this paper we propose a control scheme based on the velocity estimation using only position measurement, through a state reconstructor of first order. The estimated speed is introduced in the implicit control law , for stabilizing a two degree of freedom closed-kinematic chain mechanism. It is shown through simulations, that practically there is not any difference between the implicit control law applied directly (with data of position and velocity known), and the same law applied knowing only the position (and estimating the velocity). It is also shown that the system is asymptotically stable. The stability proof is based on the 2nd Lyapunov method.

Keywords: Parallel Robots, Lyapunov 2nd method, Implicit Systems.

Resumen

En este artículo proponemos un esquema de control basado en la estimación de la velocidad usando solamente la medición de posición, con un reconstructor de estado de primera orden. La velocidad estimada se introduce en la ley implícita de control, para estabilizar un mecanismo de una cadena cinemática cerrada de dos grados de libertad. Se muestra con simulaciones, que prácticamente no existe ninguna diferencia entre la ley implícita de control aplicada directamente (con los datos conocidos de posición y velocid), y la misma ley aplicada conociendo solamente la posición (y estimando la velocidad). También se muestra que el sistema es asimptóticamente estable. La demostración de estabilidad se basa en el segundo método de Lyapunov.

Palabras Claves: Robots Paralelos, Segundo Método de Lyapunov, Sistemas Implícitos.

1 Introduction

At the present time, robotics research puts considerable effort in the study of parallel robots because of their high precision positioning capabilities and non cumulative link errors due to their high structural rigidity. Moreover parallel robots have higher strength-to-weight ratios in comparison with conventional series manipulators (see: [Nguyen and Poorand 1989],[Fither,1986],[Lebret and Lewis 1993],[Nguyet et al 1993] and [Merlet,1990]).Such is the case of the Stewart Platform-based manipulator (SPBM) which appears simple and refined to the point of elegance. However, the same closed kinematics that provides mechanical stiffness also presents an extremely difficult theoretical problem for dynamical analysis . This problem has blocked the development of a practical control algorithm which be capable of providing a trajectory in real time, a necessity for application of the manipulator.

Although most of the research in the literature has devoted extensive effort to the kinematics, dynamics, and mechanics design of the SPBM, little attention has been paid to the control problem of this type of manipulators. [Reboulet and Pigeyre 1990] investigated the force position control of a manipulator of a six degree-of-freedom (dof) manipulator system SPBM.[Nguyen, Pooran, and Premack,1998] proposed a control scheme providing active compliance to SPBM, and presented computer simulation results of a 2-dof parallel manipulator.

Most of these control algorithms for parallel manipulators, have been developed taking in account that both the position and speed of the manipulator are known. These control design schemes vary from the very simple to the very sofisticated, and can guarantee asymptotic stability and in some cases, exponential stability in a local sense (see [Canudas and Slotine,1991],[Berghuis and Nijmeijer,1993]). The central part of these control schemes are based on the fact that the proportional and derivative control action or (PD) are known, i.e., the position and speed are available to design a linear and non linear feedback state. The state position can be measured using code-sensors, which can provide a measure of the position displacement with great accuracy [Canudas and Fixot,1991]. By contrast, the speed value which can be known using tachometers, very often can be polluted with noise or unwanted disturbances [Berghuis1992 and 193]. This problem reduces very much the manipulator's control performance. For this reazon it is important to use control schemes based on the assumption that only accure data from the position is available.

There are many research works based on the hypothesis that only available data from the position is known. Canudas and Slotine, *for* example, proposed a modified control version employing the calculated torque method, in which the velocity was replaced for a speed estimation through a non lineal observer, based on the robot pasive properties.

In this work we propose a control scheme based on the speed estimation through a state reconstructor of first order. The estimated speed is introduced in the implicit control law proposed in [Aguilar and Bonilla, 1998 and 1999]. That control law has the property that guarantees asymptotic and exponential stability when the initial conditions are close to the equilibrium point (see [Aguilar and Bonilla 1999]). Besides it guarantees the "model matching", i. e., the assignment of a dynamic behaviour to the system in closed loop through a feedback state.

This paper is organized as follows, section 2 presents the traditional closed kinematic chain, we describe there the dynamical model and discuss some important mechanical properties related to the manipulator, these properties will permit us to establish the Lyapunov function in order to guarantee the stability of the closed dynamical system. In Section 3, we describe the implicit control and proposes a state reconstructor to estimate the velocity. Section 4, is devoted to study the stability of the closed loop system using a state reconstructor. In Section 5, we present our conclusions and remarks. In section 6, we show simulations results and demonstrate the self recovering features of the states using the reconstructor. Finally, we include an appendix concernig the proof of some properties and Lemmas.

Let us finish this Section setting the following notation: $\lambda_M\{X\}$ and $\lambda_m\{X\}$ designate the maximum and minimum eigenvalue of the symmetric matrix X. Trace$\{A\}$ and det$\{A\}$ designate the trace and determinant of a given matrix A and

$$C_1 = \tfrac{x}{l_1}; \quad C_2 = \tfrac{L-x}{l_2}; \quad S_1 = \tfrac{y}{l_1}; \quad S_2 = \tfrac{y}{l_2};$$
$$\left\|[a \;\; b]^T\right\| = \sqrt{a^2+b^2}; \qquad \|A\| = \sqrt{\lambda_M\{A^TA\}}.$$

2 Fundamental Closed Kinematic Chain

Let us consider the following basic triangle chain :

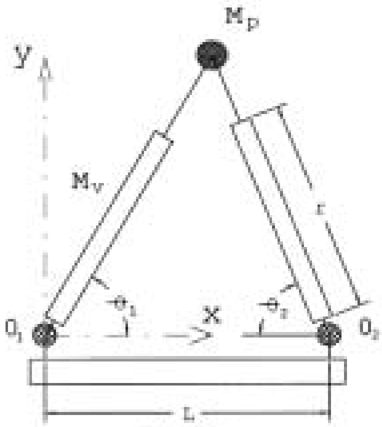

Fig. 0

This is basically constituted by two electrical pistons linked to each other at the upper extremity by a ball-and-socket joint; the other two lower extremities are linked to a fixed beam which is the base of the platform; each lower extremity in turn, is mounted to the platform by one rotatory joint. There is also a mechanical load (a mass M_p) located at the upper extremity. The left piston can move about the fixed point 0_1 whereas the right piston can move about the fixed point 0_2, and L is the fixed separation between 0_1 and 0_2 (length of the base platform). The origin of coordinates is chosen at the left piston beam joint 0_1, the $x-$axis lies at the base of the platform and the $y-$ axis points upward from the base. To define the Cartesian variables we proceed to assign two independent coordinates $x(t)$ and $y(t)$; $x(t)$ is the projection onto the $x-$ axis, and $y(t)$ is the projection onto the $y-$ axis at the same distance from the point O_p. θ_1 and θ_2 are the angles formed between the left piston and the $x-$ axis and the right piston and the $y-$axis respectively. The f_i are the forces supplied by the pistons, r is the length of the main body of the piston having a mass M_v (we have assumed that the mass is concentrated at $r/2$ and we have neglected the piston rod mass), J is the inertia moment of each actuator, and $l_i(t)$ are the variable lengths of the pistons satisfying $l_i \geq r$.

Let us express the angles θ_i in Cartesian coordinates:

$$\theta_1 = \arctan\left(\frac{y}{x}\right); \qquad \theta_2 = \arctan\left(\frac{y}{L-x}\right).$$

Let us express the lengths l_i in Cartesian coordinates also:

$$l_1 = \sqrt{x^2+y^2} \quad ; \quad l_2 = \sqrt{(L-x)^2+y^2} \ .$$

2.1 Dynamical Model

In [Aguilar and Bonilla 1998], we obtained the following model:

$$M(q)\,\ddot{q} + \Phi(q,\dot{q})\,\dot{q} + G(q) = f_q \tag{1}$$

where q is the position vector $q = \begin{bmatrix} x & y \end{bmatrix}^T$ and f_q is the force vector $\begin{bmatrix} f_x & f_y \end{bmatrix}$,

$$f_x = f_1 C_1 - f_2 C_2 \quad ; \quad f_y = f_1 S_1 + f_2 S_2; \tag{2}$$

$M(q)$ is the symmetrical inertia matrix

$$M(q) = \begin{bmatrix} M_p + J\left(\frac{S_1^2}{l_1^2} + \frac{S_2^2}{l_2^2}\right) & J\left(\frac{C_2 S_2}{l_2^2} - \frac{C_1 S_1}{l_1^2}\right) \\ J\left(\frac{C_2 S_2}{l_2^2} - \frac{C_1 S_1}{l_1^2}\right) & M_p + J\left(\frac{C_1^2}{l_1^2} + \frac{C_2^2}{l_2^2}\right) \end{bmatrix}; \tag{3}$$

$\Phi(q,\dot{q})$ is the Coriolis matrix

$$\Phi(q,\dot{q}) = J \begin{bmatrix} \frac{\delta_2 S_2^2}{l_2^2} + \frac{\delta_1 S_1^2}{l_1^2} & \frac{\delta_2 C_2 S_2}{l_2^2} - \frac{\delta_1 C_1 S_1}{l_1^2} \\ \frac{\delta_2 C_2 S_2}{l_2^2} - \frac{\delta_1 C_1 S_1}{l_1^2} & \frac{\delta_2 C_2^2}{l_2^2} + \frac{\delta_1 C_1^2}{l_1^2} \end{bmatrix}; \tag{4}$$

δ_1 and δ_2 are defined as follows:

$$\delta_1 = \frac{\dot{l}_1}{l_1} \quad ; \quad \delta_2 = \frac{\dot{l}_2}{l_2} \ , \tag{5}$$

and $G(q)$ is the gravity vector:

$$G(q) = \begin{bmatrix} \frac{M_s g r}{2}\left(\frac{C_1 S_1}{l_1} + \frac{C_2 S_2}{l_2}\right) \\ M_p g + \frac{M_s g r}{2}\left(\frac{C_1 C_1}{l_1} + \frac{S_2 S_2}{l_2}\right) \end{bmatrix}. \tag{6}$$

2.2 Energy Equations

In this subsection we give important relations of potential and kinetic energies. Let E_c be the kinetic energy created by the translational motion of load mass M_p, plus the kinetic energy produced by the rotational motion of each actuator:

$$E_c = \frac{\dot{q}^T M(q) \dot{q}}{2} \ . \tag{7}$$

Let E_p the potential energy of the load mass plus the potential energy stored in each actuator:

$$E_p(q) = M_p g y + \frac{M_s g r y}{2\sqrt{x^2+y^2}} + \frac{M_s g r y}{2\sqrt{(L-x)^2+y^2}} \ . \tag{8}$$

Let us note that:

$$\int_0^t \dot{q}^T(\tau) G(q(\tau)) d\tau = E_p(q(t)) - E_p(q(0)) \ . \tag{9}$$

2.3 Mechanical Properties

In this part we establish some important properties that will be used to analize the stability of the implicit control.

In [Aguilar and Bonilla, 1993] we have proved the following properties (See Appendix for details):

P.1) $M(q)$ is a positive definite matrix.

P.2) N is a matrix defined as follows:

$$N = \dot{M}(q) - 2\Phi(q,\dot{q}). \tag{10}$$

then $y^T N(q) y = 0 \quad \forall \quad y \in R^2$.

P.3) $\exists\, \underline{\mu} > 0$ and $\mu > 0$ such that:

$$\underline{\mu} \le \lambda_m\{M(q)\} \le \|M(q)\| \le \mu;$$
$$\underline{\mu} = \frac{M_p^2}{K_M}; \quad \mu = 2M_p + \frac{4J}{r^2}.$$

P.4) $\exists\, K_G > 0$ such that:

$$\|G(z) - G(w)\| \le K_G \|z - w\|; \quad K_G = \frac{6M_v g}{r}. \tag{11}$$

3 Control Law

In [Aguilar and Bonilla, 1999], we have proposed the following Linear Implicit Control Law :

$$\dot{\chi} = \begin{bmatrix} -k_1 & 1 \\ -k_0 & 0 \end{bmatrix} \chi + B_K \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \tag{12}$$

$$\dot{\nu} = \begin{bmatrix} -k_1 & 1 \\ -k_0 & 0 \end{bmatrix} \nu + B_K \begin{bmatrix} y \\ \dot{y} \end{bmatrix}, \tag{13}$$

$$f_q = \frac{\mu}{\varepsilon} \begin{bmatrix} \chi_1 \\ \nu_1 \end{bmatrix} - \mu k_0 q - k_{1\varepsilon}\,\dot{q} + k_{0\varepsilon} R_q + G(R_q), \tag{14}$$

where B_K is the matrix

$$B_K = \begin{bmatrix} -k_0 & -\beta \\ -k_0\beta & (k_0 - k_1\beta) \end{bmatrix},$$

R_q is the constant reference vector $R_q = \begin{bmatrix} r_x & r_y \end{bmatrix}^T$, $G(q)$ is the operator defined in (6) ,

$$k_{0\varepsilon} = \mu\frac{\beta + \varepsilon k_0}{\varepsilon} \quad k_{1\varepsilon} = \mu\frac{1 + \varepsilon k_1}{\varepsilon} \tag{15}$$

ε and β are two positive constants and k_0 and k_1 are the positive coefficients of the Hurwitz polynomial $\lambda^2 + k_1\lambda + k_0$.

Remark 1 *χ and ν are implicits variables that guarantee the linear model matching accurately when $\varepsilon \to 0$. That is, we assign to the non linear system a preestablished linear dynamics of second order. For the case that $\varepsilon > 0$, we can find a $\varepsilon^* > 0$ (which in turn depends of the states initial conditions) such that, for all $\varepsilon \in (0, \varepsilon^*)$ the closed loop system behaves like a linear system of second order. For more details see [Aguilar and Bonilla, 1999].*

4 State Reconstructor

In this section we are to assume that the state $\dot{q}$ (that is, the velocity) is not available and we will take into account the state reconstructor proposed in [Bonilla and Malabre, 1995]. Let us first propose the following proper filter:

$$\varepsilon_0 \dot{\hat{q}} + \hat{q} = q. \tag{16}$$

where the vector $\hat{q} = [\hat{x} \;\; \hat{y}]^T$ is the estimated state and $\varepsilon_0 > 0$.

Let us next substitute q by $\hat{q}$ in the I.C.L. (equations 12 to 14), then we have:

$$\dot{\chi} = \begin{bmatrix} -k_1 & 1 \\ -k_0 & 0 \end{bmatrix} \chi + B_K \begin{bmatrix} \hat{x} - r_x \\ \dot{\hat{x}} \end{bmatrix}, \tag{17}$$

$$\dot{\nu} = \begin{bmatrix} -k_1 & 1 \\ -k_0 & 0 \end{bmatrix} \nu + B_K \begin{bmatrix} \hat{y} - r_y \\ \dot{\hat{y}} \end{bmatrix}, \tag{18}$$

$$f_q = \frac{\mu}{\varepsilon} \begin{bmatrix} \chi_1 \\ \nu_1 \end{bmatrix} - \mu k_0(\hat{q} - R_q) - k_{1\varepsilon} \dot{\hat{q}} + G(R_q), \tag{19}$$

Note that this control law can also be expressed as follows:

$$f_q = -k_{0\varepsilon}(\hat{q} - R_q) - k_{1\varepsilon} \dot{\hat{q}} + \frac{\mu \hat{\Omega}}{\varepsilon} + G(R_q), \tag{20}$$

where $\hat{\Omega} = \begin{bmatrix} \chi_1 + \beta(\hat{x} - r_x) & \nu_1 + \beta(\hat{y} - r_y) \end{bmatrix}^T$, is the solution of the following ordinary vectorial differential equation:

$$\ddot{\hat{\Omega}} + k_1 \dot{\hat{\Omega}} + k_0 \hat{\Omega} = 0. \tag{21}$$

Let us express (21) by the following state space notation:

$$\frac{d}{dt} \begin{bmatrix} \hat{\Omega} \\ \dot{\hat{\Omega}} \end{bmatrix} = A_\Omega \begin{bmatrix} \hat{\Omega} \\ \dot{\hat{\Omega}} \end{bmatrix}, \tag{22}$$

where

$$A_\Omega = \begin{bmatrix} 0 & I_2 \\ -k_0 I_2 & -k_1 I_2 \end{bmatrix}, \tag{23}$$

Since A_Ω is exponentially stable, there exists a positive definite matrix P_1 such that the following Lyapunov equation is fulfilled

$$A_\Omega^T P_1 + P_1 A_\Omega = -I_{4\times 4} \tag{24}$$

where

$$P_1 = \begin{bmatrix} \left(\frac{1}{2k_1} + \frac{k_0 + k_1^2}{2k_0 k_1}\right) I_2 & \frac{I_2}{2k_0} \\ \frac{I_2}{2k_0} & \left(\frac{1}{2k_1} + \frac{1}{2k_0 k_1}\right) I_2 \end{bmatrix} > 0. \tag{25}$$

5 Asymptotic Stability

In this section we study the stability of system (1) for the case when there is feedback caused by the implicit control law(I.C.L.), see equations (16-19). Using Lyapunov 's second method we show that if the positive coefficients ε and ε_0 of the I.C.L are chosen less than a specific bound, we can guarantee that the closed loop system is asymptotically stable.

Substituting the control law (20) into system (1) we get the following closed-loop description:

$$\begin{aligned} &\dot{\hat{q}} = \tfrac{q - \hat{q}}{\varepsilon_0}; \\ &M(q)\ddot{q} + \Phi(q, \dot{q})\dot{q} + G(q) = \tfrac{\mu \hat{\Omega}}{\varepsilon} \\ &+ G(R_q) - k_{0\varepsilon}(\hat{q} - R_q) - k_{1\varepsilon}\dot{\hat{q}} \end{aligned} \tag{26}$$

and let us define ϖ as:

$$\overline{\varpi} = \begin{bmatrix} (q - R_q)^T & \dot{q}^T & \Omega^T & \dot{\Omega}^T & (\hat{q} - R_q)^T \end{bmatrix}^T.$$

Theorem 1 *The closed loop system (26) is asymptotically stable (AS) if*

$$\begin{aligned} A.1 &: \quad \overline{k}_\varepsilon > \frac{\mu}{2\varepsilon} + \frac{\varepsilon_0 \mu}{2\varepsilon}, \quad \varepsilon_0 < \min\{2, \lambda_m\{P\}\} \\ A.2 &: \quad \frac{1}{\varepsilon} > \{\frac{K_G}{\beta\mu}, \frac{1}{\beta\mu}\} \end{aligned}$$

where

$$\overline{k}_\varepsilon = k_{1\varepsilon} - \varepsilon_0 k_{0\varepsilon} \tag{27}$$

Proof: This Theorem will be proven in four steps. First, we present the Lyapunov function and we show that is definite positive. Next, we compute the derivative of the Lyapunov function. Then we show that it is negative semi-definite. At the end, we study the asymptotically stability of the closed loop system.

First step: we first introduce the following Lyapunov function

$$\hat{V}(\overline{\varpi}) = \hat{V}_1(\overline{\varpi}) + \hat{V}_2(\overline{\varpi}), \tag{28}$$

where

$$\begin{aligned} &\hat{V}_1(\overline{\varpi}) = V_G(q) + \beta_0 + V_1(\dot{q}) + V_2(\hat{q}); \\ &\hat{V}_2(\overline{\varpi}) = \tfrac{\varepsilon_0 \overline{k}_\varepsilon \dot{\hat{q}}^T \dot{\hat{q}}}{2} - \tfrac{\mu\varepsilon_0 \dot{\hat{q}}^T \hat{\Omega}}{\varepsilon} + V_3(\hat{\Omega}); \end{aligned} \tag{29}$$

V_G is the potential gravity energy,

$$V_G(q) = E_p(q) - E_p(R_q) - G^T(R_q)(q - q(0)), \tag{30}$$

which satisfies (see the Lemma A.2 in the appendix for the proof):

$$\begin{aligned} &-\beta_0 - \tfrac{\|q - R_q\|^2}{2} \leq V_G(q); \quad \text{and} \\ &\beta_0 = 2M_v g r + K_g \|R_q - q(0)\| + \tfrac{(M_v g + K_g)^2}{2} \end{aligned} \tag{31}$$

where

$$
\begin{array}{l}
V_1(\dot{q}) = \frac{\dot{q}^T M(q)\dot{q}}{2}; \quad V_2(\widehat{q}) = \frac{k_{0\varepsilon}\|\widehat{q}-R_q\|}{2} \\
V_3(\widehat{\Omega}) = \frac{\mu}{2\varepsilon}\left[\begin{array}{cc} \widehat{\Omega}^T & \dot{\widehat{\Omega}}^T \end{array}\right] P \left[\begin{array}{c} \widehat{\Omega}^T \\ \dot{\widehat{\Omega}}^T \end{array}\right],
\end{array} \tag{32}
$$

$$
P = \left[\begin{array}{cc} P_1 & 0_{2\times 2} \\ 0_{2\times 2} & P_1 \end{array}\right]. \tag{33}
$$

We next show that this Lyapunov function is definite positive. Noting that by the condition *A.2* and the definition of $k_{0\varepsilon}$, we clearly have that $k_{0\varepsilon} = \mu(\beta+\varepsilon k_0)/\varepsilon > 1$ and from (31), it follows that:

$$
\widehat{V}_1(\varpi) \geq \frac{k_{0\varepsilon}\|q-R_q\|^2}{2} - \frac{\|q-R_q\|^2}{2} \geq \frac{\|q-R_q\|^2}{2},
$$

therefore $\widehat{V}_1(\varpi) \geq 0$.

On the other hand, if $\lambda_m\{P\}/2 > \varepsilon_0$ then $\widehat{V}_2(\varpi) > 0$. Indeed from $\widehat{V}_2$, we have:

$$
\begin{array}{l}
\widehat{V}_2(\varpi) = \frac{\varepsilon_0 \overline{k}_\varepsilon \dot{\widehat{q}}^T \dot{\widehat{q}}}{2} - \frac{\mu\varepsilon_0 \dot{\widehat{q}}^T \widehat{\Omega}}{\varepsilon} + V_3(\widehat{\Omega}) \geq \\
\frac{\varepsilon_0 \overline{k}_\varepsilon \left\|\dot{\widehat{q}}\right\|^2}{2} - \frac{\mu\varepsilon_0\left(\left\|\dot{\widehat{q}}\right\|^2 + \|\widehat{\Omega}\|^2\right)}{2\varepsilon} + \frac{\mu\lambda_m\{P\}\|\widehat{\Omega}\|^2}{2\varepsilon} \geq 0.
\end{array}
$$

In a word, $\widehat{V}(\varpi) = \widehat{V}_1(\varpi) + \widehat{V}_2(\varpi) > 0$.

Second step: The derivative of $\widehat{V}(\varpi) = \widehat{V}_1(\varpi) + \widehat{V}_2(\varpi)$, is computed as follows; the derivative of $\widehat{V}_1(\varpi)$, is:

$$
\dot{\widehat{V}}_1(\varpi), = -k_{1\varepsilon}\, \dot{q}^T \dot{\widehat{q}} - k_{0\varepsilon}\, \dot{q}^T\, (\widehat{q} - R_q). \tag{34}
$$

And the derivative of each term of $\dot{\widehat{V}}_2$,is given as:

$$
\begin{array}{l}
\dot{V}_2\,(q) = \varepsilon_0 k_{0\varepsilon}\, \dot{q}^T \dot{\widehat{q}} + k_{0\varepsilon}\, \dot{q}^T\, (\widehat{q} - R); \\
\frac{d}{dt}\left(\frac{\varepsilon_0 \overline{k}_\varepsilon \dot{\widehat{q}}^T \dot{\widehat{q}}}{2}\right) = \overline{k}_\varepsilon\, \dot{q}^T \dot{\widehat{q}} - \overline{k}_\varepsilon \left\|\dot{\widehat{q}}\right\|^2; \\
\frac{d}{dt}\left(-\frac{\mu\varepsilon_0 \dot{\widehat{q}}^T \widehat{\Omega}}{\varepsilon}\right) = -\frac{\mu\varepsilon_0}{\varepsilon}\left(\frac{\dot{q}^T \widehat{\Omega} - \dot{\widehat{q}}^T \widehat{\Omega}}{\varepsilon_0} + \frac{\dot{\widehat{q}}^T \dot{\widehat{\Omega}}}{\varepsilon_0}\right); \\
\dot{V}_3\,(\widehat{\Omega}) = -\frac{\mu\left(\|\widehat{\Omega}\|^2 + \left\|\dot{\widehat{\Omega}}\right\|^2\right)}{\varepsilon}
\end{array} \tag{35}
$$

Finally, substituting (34) and each term of (35) into the expression of $\dot{\widehat{V}}$, we have

$$
\dot{\widehat{V}}\,(\varpi) = -\overline{k}_\varepsilon \left\|\dot{\widehat{q}}\right\|^2 + \frac{\mu \dot{\widehat{q}}^T \widehat{\Omega}}{\varepsilon} - \frac{\varepsilon_0 \mu \dot{\widehat{q}}^T \dot{\widehat{\Omega}}}{\varepsilon} - \frac{\mu\left(\|\widehat{\Omega}\|^2 + \left\|\dot{\widehat{\Omega}}\right\|^2\right)}{\varepsilon}. \tag{36}
$$

Third step: We find an upper bound for $\dot{\widehat{V}}$. For this we apply the following inequality $2ab \leq (a^2 + b^2)$ into the last equality, to obtain

$$
\begin{array}{l}
\dot{\widehat{V}}\,(\varpi) \leq -\left(\overline{k}_\varepsilon + \frac{\mu}{2\varepsilon} + \frac{\varepsilon_0 \mu}{2\varepsilon}\right) \left\|\dot{\widehat{q}}\right\|^2 - \frac{\mu\|\widehat{\Omega}\|^2}{2\varepsilon} \\
-\left(\frac{\mu}{\varepsilon} - \frac{\varepsilon_0 \mu}{2\varepsilon}\right) \left\|\dot{\widehat{\Omega}}\right\|^2
\end{array}, \tag{37}
$$

using *A.1* we have $\dot{\widehat{V}}\,(\varpi) \leq 0$.

Fourth step: Since $\widehat{V}$ is positive definite and $\dot{\widehat{V}}$ is only negative semidefinite, we have only proved stability in the sense of Lyapunov, namely, that the error and velocities are bounded. The $\mathcal{AS}$ of the equilibrium point $\overline{\varpi}_e = (q^T = R_q^T, \dot{q} = 0, \widehat{\Omega}^T = 0, \dot{\widehat{\Omega}}^T = 0, \widehat{q}^T = R_q^T)$ follows from LaSalles's Theorem. Indeed, let us first note from (28) to (33), that $\widehat{V}(\varpi)$ is radially unbounded, let us next define the following maximal invariant set:

$$
\begin{array}{rcl}
S & = & \left\{\varpi \in R^{10} \left| \dot{\widehat{V}}\,(\varpi) = 0 \right.\right\} = \\
 & & \left\{\varpi \in R^{10} \left| \varpi = \left[\begin{array}{ccccc} c^T & 0 & 0 & 0 & c^T \end{array}\right] \right.\right\},
\end{array}
$$

and let us finally take any trayectory ϖ belonging to S for all t, i.e.,

$$
q(t) = c;\ \dot{q}\,(t) = 0;\ \widehat{\Omega}(t) = 0;\ \dot{\widehat{\Omega}}\,(t) = 0;\ \widehat{q}(t) = c,
$$

also by the second equation of (26), we obtain

$$
k_{0\varepsilon}\,\|c - R_q\| = \|G(c) - G(R_q)\| \leq K_G\,\|c - R_q\|.
$$

From *A.2*, we have that $k_{0\varepsilon} > K_G$ and substituting this condition into the last inequality, we must have that $c = R_q$. Therefore the only one solution that can stay in S for all t is the equilibrium point

$$
\overline{\varpi}_e = (q^T = R_q^T, \dot{q} = 0, \widehat{\Omega}^T = 0, \dot{\widehat{\Omega}}^T = 0, \widehat{q}^T = R_q^T).
$$

Thus the closed loop system is $\mathcal{AS}$. □

Remark 2 *$\widehat{V}_1(\varpi)$ represents the sum of the potential and kinetic energies of the mecanical system. $\widehat{V}_2(\varpi)$ is the sum of the kinetic energy associated with the estimated state $\dot{\widehat{q}}$, plus the energy associated with the implicit variable $\widehat{\Omega}$, plus the interations between the states $\widehat{\Omega}$ and $\widehat{q}$.*

Corollary 2 *Under the same conditions as Theorem 1, the implicit control law (16)-(19) is $\mathcal{AS}$ with respect to the equilibrium point $\overline{\varpi}_e$.*

Proof: Let us first prove that the signals χ and ν, given in the Implicit Control Law are $\mathcal{AS}$.

Let us note that the equations (17) are equivalent to the following two differential equations

$$
\begin{array}{l}
\ddot{\chi}_1 + k_1\, \dot{\chi}_1 + k_0\chi_1 = (-\beta\, \ddot{\widehat{x}} + k_1\, \dot{\widehat{x}} + k_0(\widehat{x} - r_x)), \\
\ddot{\chi}_2 + k_1\, \dot{\chi}_2 + k_0\chi_2 = (k_0 - k_1\beta)(\ddot{\widehat{x}} + k_1\, \dot{\widehat{x}} + k_0(\widehat{x} - r_x)).
\end{array}
$$

Defining the auxiliary variables $\Omega_x = \chi_1 + \beta(\widehat{x} - r_x)$ and $\Psi_x = (k_0 - k_1\beta)(\widehat{x} - r_x) - \chi_2$,[1] we can rewrite the last

[1]Recalling that $q = [x, y]$ and $R_q = [r_x, r_y]$.

two equations as

$$\ddot{\Omega}_x + k_1 \dot{\Omega}_x + k_0 \Omega_x = 0,$$
$$\ddot{\Psi}_x + k_1 \dot{\Psi}_x + k_0 \Psi_x = 0.$$

From the above equations we note that Ω_x and Ψ_x are Hurtwitz and by Theorem 1, we have that $\widehat{q}$ and $\dot{\widehat{q}}$,are bounded and satisfies:

$$\lim_{t\to\infty} \widehat{x} = r_x, \lim_{t\to\infty} \dot{\widehat{x}} = 0, \lim_{t\to\infty} \widehat{y} = r_y, \lim_{t\to\infty} \dot{\widehat{y}} = 0$$

Then by definition of Ω_x and Ψ_x, we have that χ_1 and χ_2 are $\mathcal{AS}$. *i.e.*:

$$\lim_{t\to\infty} \chi_1 = 0; \quad \lim_{t\to\infty} \chi_2 = 0.$$

In the same way, we show that ν is $\mathcal{AS}$.

Let us finally prove that f_q is $\mathcal{AS}$ with respect to the equilibrium point $\overline{\omega}_e$. Since the variables χ, ν, q, $\widehat{q}$ and $\dot{\widehat{q}}$ are $\mathcal{AS}$, then from (19), we conclude that f_q is bounded and fulfilled:

$$\lim_{t\to\infty} f_q = G(R_q)$$

In summary all the internal signals of the implicit control law are bounded and converge in asymptotical way to its equilibrium point. □

6 Concluding Remarks

In this paper we have proposed a state reconstructor using a filter to estimate the position and velocity of the closed kinematic chain . This estimated state is used as input of the implicit control law which aim is to stabilize a two degree of freedom parallel manipulator. The state reconstructor depends on the parameter ε_0 and the implicit control needs to adjust only four parameters, namely ε, β, k_0, and k_1. The constants ε_0 and ε determine the stability of the closed loop system (see Theorem 1) , β determines the smoothness of the control action (see (15)), and the constants k_0 and k_1 determine the dynamics of the implicit action (see (17 to 19)).

Note that Theorem 1 states that the system is $\mathcal{AS}$ when the parameter ε_0 is less than $\lambda_m\{P\}$ and the parameter ε is less than a certain bound which is directly related with the Lipschitz constant K_G (see P.4); namely the $\mathcal{AS}$ is directly related to the weight of the linear density of the action load.

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7 Simulations

In this section we will show several digital simulations of the manipulator's dynamic model described in equations 1 to 6; note that we begin the analysis applying the implicit control law -equations (12) to (15) (without estimator, i.e., with data of position and velocity known)-, and then applying the same law using only the position, and estimating the velocity through the state reconstructor (see equations 17 to 21).

The following mechanical parameters and initial conditions were considered

$$\begin{array}{ccc} M_p = 5Kg & M_v = 1Kg & r = 1m \\ L = 1.7m & x(0) = 0.8m & y(0) = 1.2m \\ \dot{x}(0) = 0.1 & & \dot{y}(0) = 0.1m; \end{array}$$

with the following control parameters and relationships given by:

$$k_0 = 1, \quad k_1 = 1, \quad \varepsilon = 0.5, \quad \varepsilon_0 = 0.05,$$
$$\beta = 1, \quad x_d = 1.8m, \quad y_d = 1.8m.$$

Furthermore $q = [x\ , y]$ and $f_q = [f_x, f_y]$.

In figure 1 and 2, it is shown the position x and y respectively, using the speed estimator (dotted line) and without it (continuous line).

In figure 3 and 4, it is shown the speed behaviour in the direction x and y respectively, using the velocity estimator (dotted line) and without it (continuous line).

In figure 5 and 6, it is shown the behaviour of the action of control f_x and f_y respectively, again using the velocity estimator (dotted line) and without it (continuous line) in Newtons. From these figures, it is clearly evident that from time $t = 6\ s.$, practically there is not any difference between the implicit control law applied directly (without estimator, that is, with data of position and velocity known), and the same law applied with the velocity estimador using only the position.

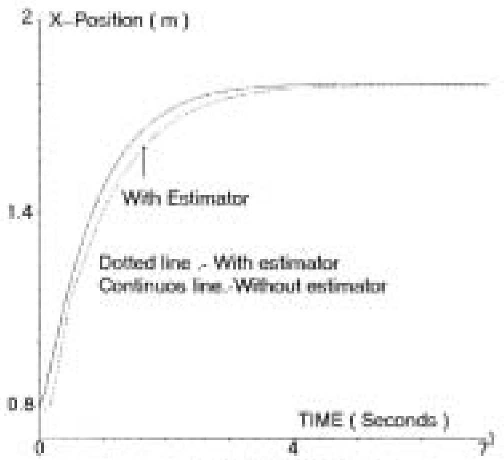

Fig. 1;position x.

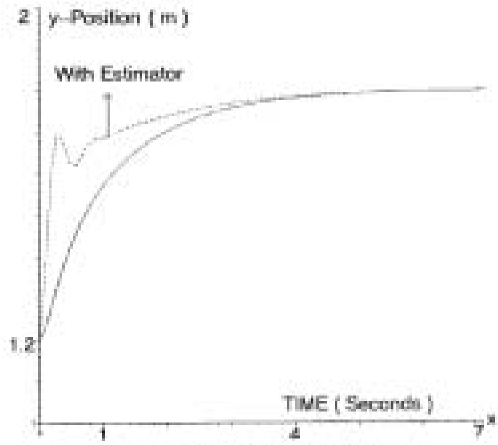

Fig. 2; position y.

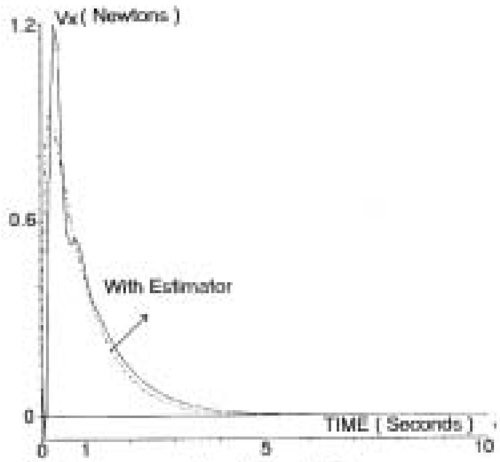

Fig. 3; velocity x.

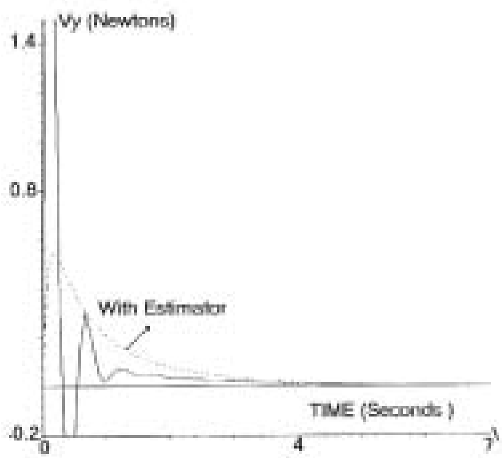

Fig. 4; velocity y.

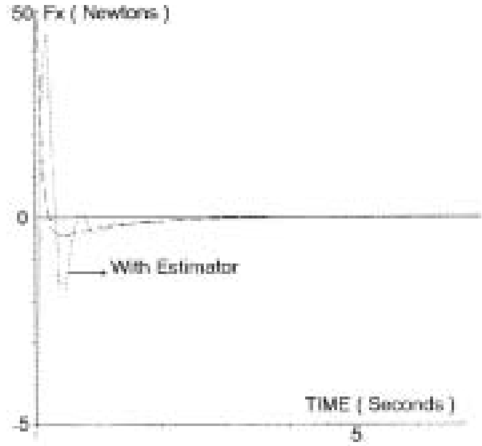

Fig. 5; control f_x.

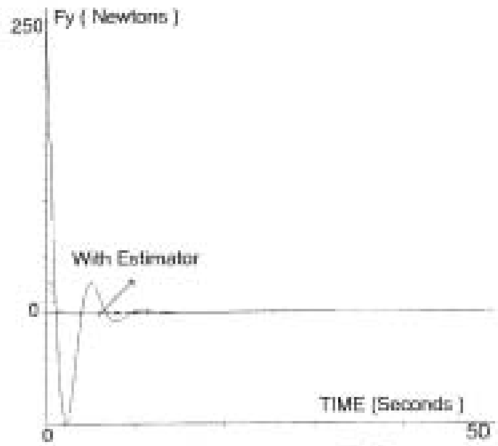

Fig. 6; control f_y.

8 Appendix

8.1 Proof of statements of section 2

Proof of (P.1): We stand for $M(q) = M_q$ By computing the determinant of $M(q)$, from (3) we have (recall that $l_i \geq r > 0$, with $i = 1, 2$ and $\forall\ t \geq 0$):

$$\det(M_q) = \\ M_p^2 + \frac{JM_p(l_1^2+l_2^2)}{l_1^2 l_2^2} + \frac{J^2(1-Cos(2(\theta_1+\theta_2)))}{2l_1^2 l_2^2} > 0 \tag{38}$$

and, since the diagonal elements are positive, we conclude the positiveness of $M(.)$. □

Proof of (P.2): In view that: 1) $M(q)$ is a positive definite matrix; 2) The kinetic energy is expressed in quadratic form in terms of $\dot{q}$, see (7); and 3) The potential energy is independent of q (see (8)) , then have, that $y^T N y = 0$, $\forall$, $y \in R^2$; therefore, N is skew symmetric. □

Proof of (P.3): Let us first compute the upper bound of $\|M(q)\|$. From (3), we have:

$$\|M_q\| = \lambda_M \{M_q\} \leq \text{Trace}\{M_q\} \leq 2M_p + \frac{4J}{r^2}$$

Let us next compute $\lambda_m\{M_q\}$. From above, we have

$$M_p^2 \le \lambda_M\{M_q\}\lambda_m\{M_q\} \le \lambda_m\{M_q\}\,\mathrm{Trace}\{M_q\}$$

and then

$$\frac{M_p^2}{\mu} \le \frac{M_p^2}{\mathrm{Trace}\{M(q)\}} \le \lambda_m\{M_q\}$$

□

Proof of (P.4): Setting $G(q) = (G_x, G_y)^T$ and using the fact that $1/l_1(t) < 1/r$ and $1/l_2 < 1/r$, from Eq.(6) one can get that:

$$\max_{x,y\neq 0} \|\nabla G(x,y)\| \le K_G.$$

Thus, we have proved that $\|\nabla G\|$ is bounded. Also, applying the mean value Theorem we can show that: $\left\|\nabla G(\tilde{x},\tilde{y})(z-w)\right\| \le K_G\|z-w\|$ for $(\tilde{x},\tilde{y}) = z\alpha + (1-\alpha)w$ and $0<\alpha<1$. The second inequality is proved in the same way. □

8.2 Proof of statements of section 4

In this subsection we prove the inequality of Eq.(31). For this we need the following Lemma.
Lemma A.1: The following inequality is fulfilled:

$$|E_p(q_1) - E_p(q_2)| \le 2M_v gr + M_p g\|q_1 - q_2\| \tag{39}$$

Proof : Let $q_i = (x_i, y_i)$, $i = 1, 2$, from (8), we have:

$$|E_p(q_1) - E_p(q_2)| \le \quad M_p g|y_1 - y_2| + 2M_v gr. \tag{40}$$

Substituting

$$\left|y_i/\sqrt{x_i^2+y_i^2}\right| \le 1 \quad \left|y_i/\sqrt{(L-x_i)^2+y_i^2}\right| \le 1,$$

into (40), we obtain:

$$|E_p(q_1) - E_p(q_2)| \le 2M_v gr + M_p g\|q_1 - q_2\|$$

□

Lemma A.2: Let us consider V_G defined in (30), then we guarantee:

$$V_G(q) \ge -\beta_0 - \frac{\|q-R_q\|^2}{2}$$
$$\beta_0 = 2M_v gr + K_g\|R_q - q(0)\| + \frac{(M_p g + K_g)^2}{2}$$

Proof : From V_G (see (30)) and using *P.4* and *Lemma A.1*, we have:

$$\begin{array}{l} V_G(q) \ge \\ -2M_v gr - M_p g\|q-R_q\| - K_g\|q - q(0)\| \end{array} \tag{41}$$

Substituting;

$$\|q(t) - q(0)\| \ge \|q(t) - R_q\| + \|R_q - q(0)\|;$$
$$(M_p g + K_g)\|q - R_q\| \le \frac{\|q-R_q\|^2}{2} + \frac{(M_p g + K_g)^2}{2};$$

into (41), we can estimate a bound for β_0 . □

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