Computing Polynomial Segmentation through Radial Surface Representation

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***Abstract*—The Visual Information Retrieval (VIR) area requires robust implementations achieved trough mathematical representations for images or data sets. The implementation of a mathematical modeling goes from the corpus image selection, an appropriate descriptor method, a segmentation approach and the similarity metric implementation whose are treated as VIR elements. The goal of this research is to found an appropriate modeling to explain how its items can be represented to achieve a better performance in VIR implementations. A direct method is tested with a subspace arrangement approach. The General Principal Component Analysis (GPCA) is modified inside its segmentation process. Initially, a corpus data sample is tested, the descriptor of RGB colors is implemented to obtain a three dimensional description of image data. Then a selection of radial basis function is achieved to improve the similarity metric implemented. It is concluded that a better performance can be achieved applying powerful extraction methods in visual image retrieval (VIR) designs based in a mathematical formulation. The results lead to design VIR systems with high level of performance based in radial basis functions and polynomial segmentations for handling data sets.**

***Index Terms*—Subspace arrangement, data modeling, segmentation, polynomial function, radial basis surface representation.**

I. Introduction

THIS paper presents an improvement in content-based applications where visual information retrieval area requires formalization methods obtaining higher implementation performance. With approximately 100 % of retrieval cases, there is not yet a methodological formalism to design visual information retrieval systems. Implementations of visual information retrieval (VIR) systems imply items as a feature extraction method, a segmentation technique, and a similarity metric. Those items have no formal or mathematical implementation framework. The formal implementation can achieve an ideal design that assures a high performance a priori. No mathematical framework has been introduced into VIR systems design, because differences between methods and features of the image collections, as well as relative lack of implementation standards and applications requirements. Today, the last implementations in segmentation approaches have to become effectives and VIR items can be attached with mathematical formalism allowed by segmentation techniques and algebraic methods to data manipulation [4]. The efficiency between image retrieval metrics and organization data methods has not been formalized [20]. With the intention to formalize the design of VIR applications and to increasing performance into the design process, we have tested approximately one thousand of images from four different libraries used for retrieving tasks matching user queries versus image collections, modeling data distribution with radial basis function, testing five different approaches for polynomial representations, the segmentation approach responsible to cluster images in groups are classified by GPCA-MVT (robust generalized principal component analysis with multivariate timing) algorithm. GPCA-MVT is selected from three varieties of it. This paper uses 477 image data classified into tree subsets with a tridimensional representation from a polynomial function computed by thin plate spline radial basis function. The surface representation improves the GPCA-MVT with the substitution of radial basis and the substitution of data collection. The final result allows proposing a mathematical expression with the possibility of to measure the performance of VIR systems from its implementation phase.

A. Mathematical Formalization Problem

The rapidly increasing power of computers has led to the development of novel applications such as multimedia, image and video databases [1]. These applications take advantage of the increasing processing power and storage of computers, which rapidly process large amounts of data. The challenge has now become one of developing suitable tools and methods for manipulation of the available data. Given the enormous amount of the information contained in a multimedia data stream, it is reasonable to consider that a deeper comprehension of the data stream may need to be achieved through the integration of independent analysis of different aspects [2], [3]. VIR techniques are an astringent need to solve several problems related to the management of these multimedia data stream. There exist several studies dealing with visual information retrieval system such as color, texture or shape [21]. The content-based visual information retrieval systems can to obtain features describing the image in more detail.

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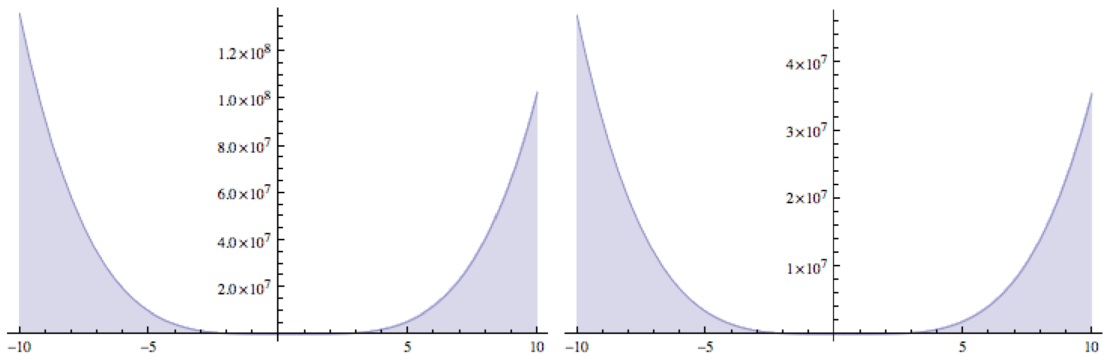

Fig. 1. Polynomial representation with $(x, y, z) =$ (Red, Green, Zero) and $(x, y, z) =$ (Red, Blue, Zero)

However, no standard has yet been formally established about:

- VIR's elements structure
- Relations between VIR's elements
- Restrictions of VIR's elements

The objective of modeling is to increase performance for VIR systems based on content-based image retrieval applications with mathematical abstraction. Practical implications referring to precision, recall and other performance evaluation metrics remain as an open problem in the field of visual information retrieval [21]. Each application content-based leads to its own ad-hoc designs, and it is valuable to know other factors involved in the process. The theoretical value of this model contributes to the mathematical handling of computer sciences within the visual information retrieval area. The VIR elements model is helpful to define constraints, parameters, standards and relations which could be helpful for content-based applications, in addition, it can be a support for systems which include high values in precision and recall [20]. Mathematical treatment implies definition of geometric dispersion and statistical analysis providing an abstract representation of data sets and improving techniques and classification results. The typical learning is statistical or probabilistic learning. Huge collections handle mixed data that are typically modeled as a group of samples $\{s_1, s_2, ..., s_n\} \in \Re$ is obtained from a learning approach with some probabilistic distribution. Each of the samples has a domain of values and they are composed of a set of vectors. Every vector can be modeled as a singular value array that describes relevant features of a sample as an image or an image collection a stellar spectrum or a document set. This approach allows retrieving images from a VIR system. With former fundamentals, this research proposes:

To design a mathematical abstraction for VIR systems design.

- To perform an analysis of a polynomial function as the representative model of data sets.
- To examine several radial basis functions as similarity metric.
- To use and modify the GPCA algorithm in basis computation.
- Finally, a mathematical formalization is proposed to encourage VIR systems efficient designs.

B. Organization of this paper

In this paper, we review the solutions to Problem 1.1 under no standards formalized yet. As a result, the scope of subjects to be covered ranging from VIR systems, from feature descriptors, and from segmentations with real data of images. Nevertheless, we hope to convince the reader that these subjects are strongly related one to another and they are crucial for researchers who want to gain a deep and complete formalization about the problem. The paper is organized as follows: Section 2 reviews the basic properties of formal polynomial representation. Section 3, we made some variation that allows us to estimate the segmentation in subspaces from sample points. Section 4, explains the mathematical formalization for VIR systems. A formula that can compute or increase the performance for content-based applications, especially for image retrieval is provided in section 5, and one explanation of how this formalization can be applied to several real-world applications is provided.

C. Polynomial Representation by Radial Basis Surface

The polynomial functions tested were useful for selection of dimensions of coil data. It is important to remember that a color descriptor was used to describe the image collection by its placement in the space. The performance of functions in two dimension or a polynomial representation and surface representation is exposed in Figure 1 and 2.

Figure 1 shows that the order of assignation (x, y) = (Red, Green) or (x, y) = (Green, Blue) is indistinct because finally, the representation is tridimensional, but a change in scales is observed for image collection polynomial representation. The coefficients are the parameter that will be useful for retrieval similarity metric.

Figure 2 indicates that surface representation is invariant to the order to assignation for (x, y) = (Red, Green) or (x, y) = (Green, Blue). The importance here is the variation

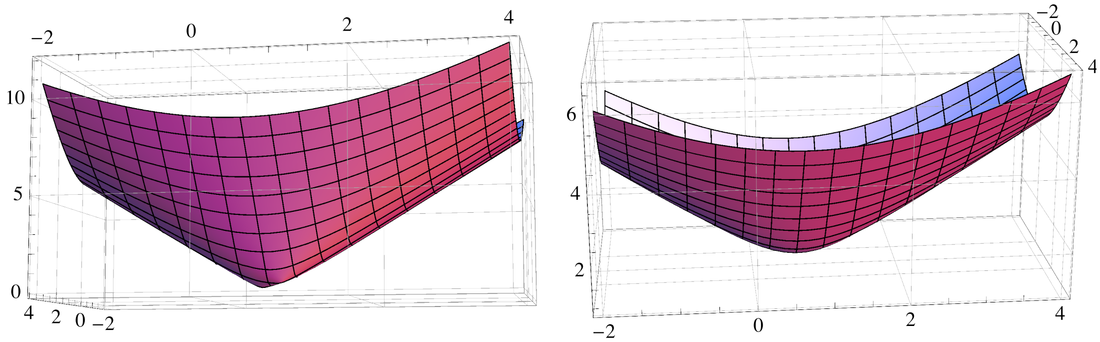

Fig. 2. Surface representation with (x, y, z) = (Red, Zero, Blue) and with (x, y, z) = (Red, Green, Zero)

of coefficients for similarity measure that are computed ad hoc to the algebraic distribution of the images or samples considered. Again, the scales of the surfaces do not change in a relevant difference. Finally, the Figure 3 implies that does not matters if assignation take care of (x, y) coordinates or if (x, z) coordinates are considered from collection data, but the only variation are the coefficients that are computed only from a representation in two dimensions.

D. Fundamentals and functions

Several works about subspace arrangements such as segmentation method in data sets have been developed. Subspace arrangements can be computed through Vanishing Ideals in polynomial rings. Some of the works detailing this process are [5], [6], and [7]. The concept of outliers in subspaces arrangements refers to elements that lie out of some data group well classified. Some papers help to detect and to avoid outliers in subspace arrangements such as [8], [10] and [11]. A variation for model detection of subspaces is described in [12], [13] and [14] as an alternative to subspace arrangements. Relevant related work deal with at GPCA (generalized components analysis) improvements as mentioned in [4], [15], [16], [17], and [18]. Recent methods and variations in linear hybrid modeling as mathematical treatments are showed in this section.

A relevant work for this research is [4], which is about a linear hybrid model based on subspace arrangements and Hilbert functions. The work [6] provides details about Hilbert function and vanishing ideal combination. The paper [15] explains details and considerations about subspace arrangements. The work [18] proposes an algebraic and geometric approach for subspace arrangements in a generalized principal component analysis. The main inspiration for this research was Yi Ma's work [4], and it is adapted to understand the mathematical formalization for VIR systems that pursue this research. A special variation of GPCA algorithm used to modeling data sets is presented in Section 1) below. The core of the similarity metric based in radial basis surface representation is explained in Section 2) below.

1) The Algorithm and the similarity metric: The original GPCA algorithm can be reviewed in [7] and the improved version in [4] for reader convenience.

2) Variation 1: Radial basis function: Given n distinct points $x_1, ..., x_n \in \Re^D$ where the function values $f(x_1, x_2, ..., x_n)$ are known $\Re^D$ are real values with dimension D, and we use an interpolate of the form:

$$s_n(x) = \sum_{i=1}^{n} \lambda_i \phi(\| x - x_i \|) + p(x), x \in \Re^D, \qquad (1)$$

where $\| . \|$ is the Euclidean norm. $\lambda_i \in \Re^D$ for $i = 1, ..., n$, with $p \in \Pi_n^d p$ (the linear space of polynomials in d variables of degree less than or equal to m), and Φ is a real valued that can take many forms. The most suitable function for this work is $\Phi(r) = r^2 log r, r > 0$ and $\Phi(0) = 0$ called surface splines [19], [20].

Let Φ be the function from surface splines:

$$\Phi(r) = \left\{ \begin{array}{ccc} r^k & k \in N, & k\ odd \\ r^k log(r) & k \in N, & k\ even \end{array} \right\} r \geq 0, \qquad (2)$$

where w is a coefficient empirically selected. Let m be any integer such that $m \geq m_\phi - 1$, and let Ω be a subset of N^d. Then we define $\mathcal{F}_{\Phi,m}(\Omega)$ and $\mathcal{A}_{\Phi,m}(\Omega)$ to be the linear function spaces. Let s and u be any functions in $\mathcal{A}_{\Phi,m}(\Omega)$. The semi-inner product is the expression:

$$\langle s, u \rangle = ((e-1)^{m_\phi}), \sum_{i=1}^{N(s)} \lambda u(y_i), \qquad (3)$$

where N is the total number of samples, lambda is the coefficients, and y_i is the output of Φ function. Thus Equation (6) is required as a semi-inner product that induces the semi-norm $\| . \| := \langle ., . \rangle^{1/2}$. The following section shows two variations of our approach. The first one is based in a basis computation of the image collection. The second variation is based in COIL-100 image collection as input data along with the RBF computation of the first variation.

3) Variation 2: Radial Basis Function as input parameter: The analysis of test for this step involves in the first place knowing the function that reveals the coefficients computed

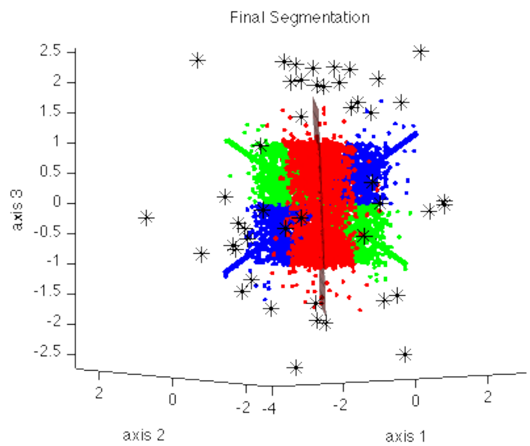

Fig. 3. Test 4 with COIL-100 as input data to RGPCA-MVT algorithm for segmentation data (see Table I)

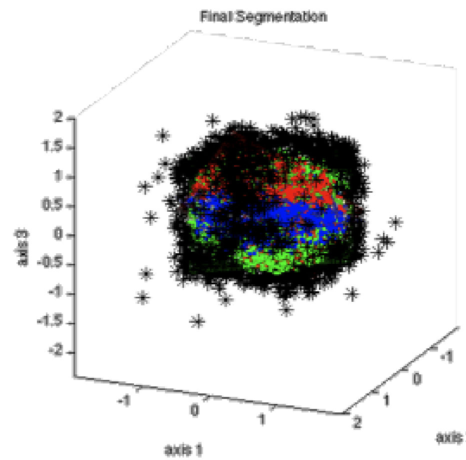

Fig. 4. The Test 12 with RBF-Basis as input data to RGPCA-MVT algorithm for segmentation data (see Table II)

for the polynomial function obtained from the best radial basis function computed as follows:

$$\varphi(x,y) = (x^2+y^2)log^{10}(\sqrt{x^2+y^2}) \tag{4}$$

The best RBF obtained was Spline, therefore, the polynomial function obtained as similarity metric measure for the first, second and third basis group for our research are computed using:

$$\begin{pmatrix} -21.3037\varphi[(x_{Q1}-x_1),(y_{Q1}-y_1)] \\ +1.58945\varphi[(x_{Q1}-x_2),(y_{Q1}-y_2)] \\ -5.39250\varphi[(x_{Q1}-x_3),(y_{Q1}-y_3)] \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} -21.3037\varphi[(x_{Q2}-x_1),(y_{Q2}-y_1)] \\ +1.58945\varphi[(x_{Q2}-x_2),(y_{Q2}-y_2)] \\ -5.39250\varphi[(x_{Q2}-x_3),(y_{Q2}-y_3)] \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} -21.3037\varphi[(x_{Q3}-x_1),(y_{Q3}-y_1)] \\ +1.58945\varphi[(x_{Q3}-x_2),(y_{Q3}-y_2)] \\ -5.39250\varphi[(x_{Q3}-x_3),(y_{Q3}-y_3)] \end{pmatrix}. \tag{7}$$

The next step implies to compute the basis for each group that is provided as input to the RGPCA-MVT algorithm. Those coefficients are computed by trial and error in the state of art, but in this case, the values are precise and established for each group of images from a random image of the group. This random image does not require be representative from the collection and does not require to be specially treated as in other works of the area. The basis for the three groups previously computed is exposed in Table I. These values are now provided as input to the RGPA-MVT algorithm. The Dimension 3 in Table I is not used for the SDM research objectives. The results obtained from RBF basis as input to the RGPCA-MVT algorithm are detailed in Table I. These results show that Average Basis Error increases, but the percentage of Error Classification is of 62.40 % (see Test 9). The classification accuracy of the segmentation obtained is 37.6 % that reflects 1 % of improvement of the RGPCA-MVT algorithm. The visual result of basis computation variation can be seen in Figure 4.

TABLE I
VALUES OF RADIAL BASIS FUNCTION PROVIDED AS BASIS INPUT FOR RGPCA-MVT ALGORITHM

Group	Dimension 1	Dimension 2	Dimension 3
1	0.131370	-0.056482	0.272775
2	0.555409	-0.002310	0.174136
3	0.749958	-0.057221	0.001160

The detected rejection rate obtained from the nine test from Table II is important only because is a warranty that at least one model is found for modeling the data collection of images, in contrast with Test 1, 2, 3, 5, 6, and 8, where no model was achieved for the segmentation data values.

TABLE II
VARIATION 2: MODIFICATION AT RGCPA-MVT ALGORITHM WITH RBF BASIS AS INPUT

Outlier Percentage	Dimension Groups	Detected Rejection	Segmentation Error	Average Basis
0.1600	(1,1,1)	No model	66.07%	81.17%
0.0800	(1,1,1)	No model	69.40%	68.60%
0.5000	(1,1,1)	No model	76.78%	65.64%
0.9000	(1,1,1)	20	66.56%	71.84%
0.1600	(2,1,1)	No model	65.31%	81.29%
0.0800	(2,1,1)	No model	68.69%	79.25%
0.5000	(2,1,1)	10	64.48%	89.70%
0.9000	(2,1,1)	No model	67.50%	76.26%
0.1600	(2,2,2)	46	62.40%	90.00%
0.0800	(2,2,2)	12	64.25%	90.00%
0.5000	(2,2,2)	35	63.26%	90.00%
0.9000	(2,2,2)	33	64.68%	90.00%

E. Mathematical Formalization with RGPCA-MVT

The main motivation of Segmentation Data Modeling (SDM) research for VIR Systems was generated starting on the following approaches: How can a mathematical model of a visual information retrieval system be constructed? The kind of modeling for visual image retrieval systems imply the previous organization of the images, along with an important and powerful algorithm like RGPCA-MVT that has been successfully used in tracking techniques [4]. This algorithm allows a segmentation of groups applying a mathematical approach to establish a plane for groups of two dimensions or more and ensures a properly image retrieval regardless disadvantages of descriptor selected. How can at least one of visual information retrieval elements be constrained? The main visual information retrieval elements are: the image collection, a feature extractor or descriptor, a similarity measure, and an organization or classification approach. Fortunately, is possible to constrain the similarity measure by radial basis function specifically by Thin Plate Splines.

Another element that can be constrained for visual image retrieval systems is the method of classification or organization of the images, and this can be constrained by the RGPCA-MVT. The other element that can be constrained is that of the image collection which must be appropriate for the application. The other item that can be constrained in a certain manner is the image descriptor. For an image descriptor it is important to mention that if the image descriptor are well selected, the classification and the retrieval can be improved. It is important to test in further work with another texture approaches. How can performance in a VIR system through a mathematical point of view of its data set be increased? The most important point of view to increase the performance in a visual information retrieval system is the requirement of a careful selection of the Descriptor containing the main feature vector extraction of the data set. How can be helpful the prediction in new data set properties for VIR systems? The data set properties are inherently algebraic, always are present in data sets and can be modeled when a feature extractor method is applied.

F. Conclusions and Perspectives

The purpose of our modeling is to increase performance for VIR systems and we found the following answers at our questions research in section 4 leading to conclude about content-based image retrieval design through a mathematical modeling. The varieties of applications in the VIR area are evaluated with certain performance metrics as recall or precision just to mention a few. The challenge of each application implies to increment of certain metrics at 100%.

Most applications never achieve 100% in recall or precision. The initial attribution were as consequence to the user subjectivity to make some kind of classifications, but now we know that there are other elements implicated that could be systematically controlled in the data sets of this kind of systems (VIR or CBIR applications) as Segmentation method, Classification percentage and Similarity metric used. Our contribution provides a framework to detect certain aspects in VIR system designs to increase percentages of performance before they are implemented or further developed.

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